SOME STOCHASTIC MODELS FOR THE STUDY OF WASTAGE IN MANPOWER PLANNING

A Thesis to the Bharathidasan University, Tiruchirappalli – 620 024, Tamil Nadu, India for the Award of Degree of

DOCTOR OF PHILOSOPHY

BY

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CERTIFICATE

This is to certify that the thesis entitled "**Some Stochastic Models for the Study of Wastage in Manpower Planning**" submitted by **Mr.A.MUTHAIYAN** for the award of the degree of Doctor of Philosophy (Ph.D) in Mathematics, to the Bharathidasan University, Tiruchirappalli, is a record of bonafide research work carried out under my supervision. He has fulfilled the requirements for the submission of the thesis, which has attained the necessary standard required for a Ph.D Degree of the University. The results presented in this thesis have not been submitted elsewhere for the award of any other Degree or Diploma.

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DECLARATION

I here by declare that the thesis entitled "**Some Stochastic Models for the Study of Wastage in Manpower Planning**" is the original work that has been carried out by me under the Supervision of **Dr.A.SULAIMAN**, Reader in Mathematics, P.G and Research Department of Mathematics, Jamal Mohamed College, Tiruchirappalli – 620 020. This thesis has not been previously formed the basis for the award of any Degree, Diploma or other similar title in any Indian or Foreign University.

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CONTENTS

Dedicated
to my
Beloved Parents

Chapter – I

Introduction

CHAPTER – I

INTRODUCTION

The social and economic well being of any country or nation is very much dependent on many factors among which the natural resources available in that country is a vital component. The social welfare of the country also depends on the economic status of that country. The economic status of that country is based on the natural resources available in that country as already indicated. Proper exploitation of the natural resources and careful utilization of the same in turn depends on the manpower availability in that country. Skilled specialized types of manpower and expertise are quite essential for achieving best results. Hence the importance of manpower and proper utilization of the same play a vital role in the economic development of any nation. In these days of fast advancement of civilization and technological changes the demand for highly specialized types of labour and manpower is unavoidable. For example the software industry is undergoing a rapid change from time to time. The constant demand for specialists in that area is felt to be very acute now a days.

There is every likelihood for the disequilibrium between the demand for manpower and the supply of the same and hence in some cases the demand may be greater than supply. In such cases the productivity of the nation or industries suffer. If there is excess of manpower than the demand for the same again it is disadvantageous in the sense that unemployment becomes inevitable. Hence ways and means of proper utilization of manpower available should be identified. It is in this context manpower planning has emerged as an important discipline under Human Resource Development (HRD). The application stochastic processes and statistical methods play a vital role in developing suitable mathematical models to portray the real life situations in manpower problems. Such models are very useful in the sense that they can be used for future projections which in turn would help policy making and decisions by the management.

According to Grinold and Marshall (1977), "Manpower Planning must be an ancient art since manpower problems have existed for centuries". It is quite appropriate to mention the definition of manpower planning as given by Walker (1968), "Manpower planning refers to the rather complex task of forecasting and planning for the right numbers and right kinds of people at the right places at right times to perform activities that will benefit both the organization and the individuals in it".

According to Grinold and Marshall (1977) "There is interaction between the four ingredients of a Manpower system which are people, jobs, time and money. The interactions among these four ingredients of a Manpower system should be taken into consideration while formulating and evaluating any manpower policy".

Early Developments in Manpower Planning

The studies in manpower management and planning have been for over four decades in the past. The use of techniques of Operations Research and Statistics have been evolved by Lane and Andrew (1955). Construction of transition models involving stocks and flows were by Young and Almond (1961) Gani (1963). The applications of renewal theory to the study of manpower planning was done by Bartholomew (1969). Application of linear programming and goal programming techniques were developed by Charnes et al (1971).

The application of stochastic process to manpower planning has put in lot of impetus to generate many stochastic models for evolving suitable manpower policies. Any real life situation can be conceptualized in terms of stochastic models for the purpose of formation of manpower policies taking into consideration the constraints if there are any. A detailed account of the different approaches to manpower planning can be had from books by Bartholomew and Morris (1971) Bartholomew and Smith (1971), Vajda. S (1975), Grinold and Marshall (1977) Bartholomew and Forbes (1979) etc.

The pressures to Improve Manpower Management

There are two different aspects associated with the manpower concepts.

1. The availability or supply,
2. The requirement or demand.

The unutilized manpower leads to loss of productivity on one hand, and non-availability of the right type of manpower for certain jobs results in adverse impact on production on the other. In addition to the above according to Malcolm Bennison and Jonathan Casson (1984), "Organizations have become more complex requiring a wider range of specialist skills in their employees. A wider range of skills means more types of employees to be recruited, and the greater the degree of difficulty in replacing such employees when they leave". The pressure on personnel and manpower specialists is high, the penalties for failure to provide and maintain appropriate policies are serious for the organization and for the individual. The conflicting demands of the organization in its search for efficiency, the individual in his desire for job security and to be recognized as a contributor, the state with its twin objectives of reducing unemployment and promoting investment, have made the manpower planning very complex.

Manpower planning has been commonly described as a process consisting of three elements:

(i) Predicting the future demand for manpower;

(ii) Predicting the future supply of manpower;

(iii) Looking at policies to reconcile any difference between the results of (i) and (ii), often known as "closing the manpower gap".

Predicting demand may involve looking at productivity changes, technological changes, market forces and trends and the corporate strategy. Predicting supply involves a knowledge of the current manpower stocks and looking at the future recruitment, wastage, working conditions, promotion policies and labour market trends. Closing the gap means examining training, remuneration, career planning, redundancies and further consideration of all the factors under other headings.

Mathematical and Stochastic models are developed using certain assumptions and taking into account the real life situations that exist. In the studies relating to manpower planning one of the important aspects of study is to derive expressions for some constants, which would help policy making in HRD. For example the leaving of personnel at random time intervals is a common phenomenon. This is also called 'wastage' in manpower studies.

Wastage

Wastage is the most fundamental aspect of manpower planning. It is as a result of individual decisions to leave the organization and so the management has very little or no control over this. The study of the concept of wastage arises due to the following.

1. In organizations where the number of jobs is controlled, wastage creates vacancies and so promotion and recruitment opportunities arise due to wastage only.
2. Manpower planning could be successful when only it is possible to predict the extent and pattern of wastage.
3. Suitable amount of recruitment could be made and promotions decided only by a proper measurement of wastage.
4. Measures of wastage are often considered as the indicators showing the health of the

organizations, the absenteeism and industrial dispute.

5. The measurement of wastage is a very important aspect. The stability of an organization is very much dependent on the quantum of wastage.

The total wastage refers to the total number of individuals who leave the system. The wastage can be voluntary or involuntary. Involuntary wastage comprises of the loss of persons from an organization due to reasons such as death, ill-health, retirement and redundancy. They are beyond the control of the individual. It is to be noted that the amount of voluntary wastage comprises of the number of losses due to the choice of the individual such as taking a job elsewhere , voluntary retirement etc.,. It cannot be precisely predicted and usually the amount of wastage due to this kind will be very small or very low compared to the other forms of wastage. Wastage patterns depend upon an individual's propensity to leave a job and it depends upon a number of factors such as personal and environmental factors. Usually much importance is given to the Completed Length of Service (CLS) that is deciding the propensity of an individual to leave the job. So, CLS distributions play a major role in wastage analysis.

Prediction of wastage is another aspect. Cohort analysis is often used for the purpose of forecasting the future wastage. The Markov chain models are also used in forecasting wastage. Measures or indices of wastage are used to compare the propensities of different groups of individuals to leave at the same time or the same group to leave at different times. The mean length of service has been proposed as a measure of stability by Lane and Andrew (1955). It is customary to represent the wastage as a row vector given by $W = (w_1, w_2, w_3, \ldots, w_k)$ known as the wastage vector.

Analysis of Wastage

In the analysis of wastage the empirical approach to the description and measurement of wastage is used and the probability models for the wastage process are considered for the summary measures and control of the wastage process. Wastage rates are obtained for a given group of individuals in some interval of time. This involves the so called cohort analysis. The completed length of service (CLS) distribution is empirically obtained using the cohort data. The survivor function which is defined as the probability that a recruit will continue in service for a length of time x is also obtained.

In modeling predictions and measurement of wastage the survivor function is given a specific mathematical form. The CLS distribution is obtained on the basis of fitting specific distributions for the completed length of service data. Many models have been studied with many underlying assumptions. Fitting of CLS distributions is an important aspect.

On every occasion of wastage the recruitment can not be carried out. Also the training of personnel recruited cannot be repeated on each occasion of recruitment. Hence the wastage on successive occasions or random epochs are allowed to accumulate. There is a particular level of manpower wastage beyond which wastage cannot be permitted. This is called the threshold. As and when the total amount of manpower wastage crosses the threshold level the recruitment of personnel becomes imperative. The expected time to cross the threshold (or) to recruitment is denoted as E(T) and V(T) is its variance, where T is a random variable which denotes the time to recruitment. In developing such models the interarrival times between random epochs of wastage, the threshold level, and the random amount of wastage at every epoch of wastage are taken to be random variables. In deriving the E(T) and V(T) the so called "shock models and cumulative damage process" by Esary Proschan and Marshall (1973) is used.

Manpower Classification

Any manpower system comprises of different classes of manpower. The total number of persons in any Organization can be partitioned into different mutually exclusive groups or classes based on the nature of jobs done by them, the rank of individuals in hierarchy, technical and non-technical cadre etc., Seniority is also an important aspect. Based on these types of classification a graded system or hierarchical order system in an organization can be conceptualized.

Stocks and Flows

In an organization with hierarchical system we have a number of classes or categories of employees. At any epoch T the number of persons in a particular class is denoted as $n_i(T)$; $i=1,2,\ldots,k$. The number of classes and $n_i(T)$ are called the stocks. Hence we have stock vector denoted as $n(T)=[n_1(T), n_2(T),..., n_k(T)]$. With the passage of time the movement of persons from one category to the other takes place and it is called flow. $N_{ij}(T-1)$ denotes the flow of number of individuals from category i to j over a period time (T-1) to (T). The flows can be represented in the from of square matrix as follows.

$$\begin{bmatrix} n_{11}(T-1) & n_{12}(T-1) & \cdots & n_{1k}(T-1) \\ n_{21}(T-1) & n_{22}(T-1) & \cdots & n_{2k}(T-1) \\ \vdots & \vdots & & \vdots \\ n_{k1}(T-1) & n_{k2}(T-1) & \cdots & n_{kk}(T-1) \end{bmatrix} = N(T-1)$$

The flows represented in the form of a matrix refer to the transfers, promotions or demotions in an organization. It may be observed that the flows in any manpower system may be due to push factors or pull factors. The persons are promoted from a lower grade to next higher grade because a vacancy arises at the higher grade in an organization and it is due to acquiring a new qualification it is called push flow. The push and pull factors play a key role in occurrence of changes in the manpower structure of the organization.

Recruitment

Recruitment generally refers to the addition of persons to the existing strength in any class or cadre. Recruitment can be either directly to a particular cadre or due to the promotion of persons from the lower cadres. Hence recruitment is a part of flow. Recruitment has an important role to play in manpower planning because the number to be recruited has to be decided from time to time taking into consideration various factors, such as cost etc., Optimal recruitment policy deals with the number to be recruited keeping in mind the costs involved.

Cost in a Manpower System

A manpower system may be compared to an inventory system in which there is demand for products and at the same time replenishment is made by the organization. In an inventory system the holding cost, reorder cost and shortage cost are taken into account while formulating an optimum inventory decision or policy. In the case of a manpower system, the depletion of manpower occurs due to wastage and the same should be compensated by replenishment of manpower. Hence recruitments are made. The optimal policy or decisions are taken by considering the associated costs. The various costs associated with policy decisions in a manpower system are given in Poornachandra Rao (1990). The following are the costs associated with a manpower system.

1. Recruitment cost
2. Cost of excess or overstaffing cost.
3. Shortage cost otherwise called understaffing cost.
4. Cost of retrenchment/retirement.
5. Retention cost.

Type of Manpower Models

Cross Sectional Model

In the study of manpower planning, the cross sectional models are widely used. In these models the stock levels of different grades of a system at a particular time point 't' is taken. The data would be on the cross sectional system at a given point of time 't'. A detailed discussion of cross sectional models is given in Grinold and Marshall (1977).

Longitudinal Model

Under these types of models the entire history of a group or cadre with regard to its size etc., are given over a period of time. It is based on historical data and such models may prove very expensive from the view point of data collection and analysis. Some times it is possible to combine both longitudinal models and cross sectional models.

Completed Length of Service (CLS)

Another important concept used in the study of manpower planning is the CLS. The CLS forms the basis for the leaving pattern or wastage process in any organization. The exponential, lognormal, Weibull distributions are all used to specify CLS. Using the CLS distribution, the survivor function may be obtained using which many other results can be derived. The empirical form of the CLS distribution can be obtained using cohort analysis. A homogeneous group of entrants is taken and the number of persons in a given cadre is observed. The group initially taken up is called the cohort. The parametric and non parametric methods of estimation of survivor function using CLS data has been attempted by several authors. The estimation of survivor function using left truncated and right censored data is given in McClean and Gribbin (1987).

Some Basic Preliminary Concepts:

The shock models deal with the life distribution of devices or components which are subjected to shocks. For example, the devices like an electric bulb, T.V. picture tube etc., are subjected to random fluctuations in the voltage of electricity. A sequence of shocks occur randomly overtime and the instantaneous damage occurring at the random epochs cumulate to an unknown threshold value beyond which the system fails. The threshold level is itself a random variable. At every shock, a random amount of damage is caused to the device and the damages in successive shocks get added together in the form of a cumulative damage. The rate at which the threshold is approached is also studied. There are various approaches to the rate of accumulation of damage, but they all appear to be resolvable in terms of a stochastic process. If the damages caused by successive shocks are i.i.d random variables denoted as X_i, $i = 1,2,3,\ldots,n$ with common distribution function F(.) and the random threshold level Y is a random variable with distribution function G(.), then the probability that the device survives k damages is denoted as

$$P_k(x) = \int_0^{\infty} F_k(x)[1 - G(x)]\, dx, k = 1,2,\cdots$$

where $F_k(x)$ is the k fold convolution F(x) with itself and $F_0(x) = 1$.

The reliability R(t) of the device is given by

$$R(t) = \sum_{k=0}^{\infty} P_k(x)\ V_k(t)$$

where $V_k(t)$ is the probability that k damages are caused during (0.t]. The above model has been considered by Esary Marshall and Proschan (1973) with the underlying process generating the shocks as Poisson.

Order Statistics

Suppose that a random sample of 'n' observations denoted as x_1, x_2, x_3,...,x_n be taken from a population which has univariate continuous distribution with p.d.f f(.) and d.f. F(.). The random sample of observations X_i, $i = 1,2,\ldots,n$ can be arranged in increasing order of magnitude such as $x_{(1)} < x_{(2)} < x_{(3)} < \ldots < x_{(n)}$. Here $x_{(1)}$ is the smallest value called the first order statistic and $x_{(n)}$ being the largest called the n^{th} order statistic. The order statistics are not independent.

For any 'r' the corresponding r^{th} order statistic is $x_{(r)}$. The distribution of the r^{th} order statistic is given by

$$f(x_{(r)}) = \frac{n!}{(r-1)!(n-r)!} F[x_{(r)}]^{r-1}$$

The distribution of the first order statistic $X_{(1)}$, is given by

$$f_{X_{(1)}}(x) = P[x_{(1)} \leq x]$$

$$= 1 - [1 - F_{(x)}]^n$$

and $f_{X_{(1)}}(x) = n[1 - F(x)]^{n-1} f(x)$

Similarly the c.d.f of the n^{th} order statistic $X_{(n)}$ is given by

$$f_{X_{(n)}}(x) = P\left[X_{(n)} \le x\right] = [F(x)]^n$$

and $f_{X_{(n)}}(x) = n[F(x)]^{n-1} f(x)$ is its p.d.f

It may be noted that in deriving E(T) and V(T), the p.d.f and d.f of $x_{(1)}$ as well as $x_{(n)}$ can be used taking $x_{(i)}$ to be the interarrival times between successive occurrences of an event.

RENEWAL THEORY:

Renewal theory arose originally from the study of self renewing aggregates and in recent years it developed substantially while investigating certain basic problems in the theory of probability connected with sums of independent, non-negative random variables. It originated with the study of certain probability problems connected with the failure and replacement of components, such as electric bulbs, industrial machinery etc: later it became clear that essentially the same type of problems arose in connection with many other applications. It was also felt that the fundamental mathematical theorems of renewal theory are of intrinsic interest in the theory of probability. With the current surge of interest in the mathematical problems of Operations Research, this theory attained added importance.

These developments can be classified under three heads:

1. Renewal theorems for a sequence of mutually independent non-negative random variables.
2. Renewal theorems for a sequence of mutually independent random variables which are not necessarily non-negative.
3. Renewal theorems for dependent random variables.

Consider a machine in which when a component fails, it is replaced by a new component. It is assumed that the components live and die independently of each other. The life time of each component is assumed to be a random variable having the distribution function

$F(x)$. Observing from the time $t = 0$, the component in question at that instant has attained the age x_0. Then the residual lifetime x_1 of this component has a distribution function given by

$$K(x) = \frac{F(x + x_0) - F(x_0)}{1 - F(x_0)}$$

If $x_0 = 0$, $K(x) = F(x)$; otherwise $K(x) \neq F(x)$. Thus the first replacement of the component occurs at time $t = x_1$ where x_1 has the distribution function $F(x)$. If the second component dies at age X_2 then the next replacement occurs at $t = x_1 + x_2$, and in this way the process continues. The process $\{x_n\}$ is called a Delayed Renewal Process, if $x_0 \neq 0$.' Thus the above process, $\{x_n\}$ $n = 1,2,...$ represents a sequence of mutually independent random variables, which assumes non-negative values in which the distribution function of x_1 is $K(x)$ and that of x_n $(n = 2,3,....)$ is $F(x)$, $(0 < x < \infty)$. We ignore the trivial case when $P(X_n = 0) = 1$. If the possible values of x_n are equal to nd, (n =

0,1,2,......) where d > 0, (d fixed), we say that the process $\{X_n\}$ is discrete, otherwise it is continuous.

Let

$$S_0 = 0$$

$$S_n = X_1 + X_2 + \ldots + X_n, \ (n \geq 1)$$

$$F_n(x) = P(S_n \leq x), \ 0 \leq x \leq \infty, \ n = 0,1,2,\ldots\ldots$$

For x > 0, defines the random variables

$$N(x) = \max \{n; s_n \leq x\}.$$

It can be easily seen that N(x) is a proper random variable with finite moments of all orders. The distribution of N(x) is given by

$P_n(x) = P[N(x) = n] = F_n(x) - F_{n+1}(x)$, n = 0,1,2,.... and its mean by

$$H(x) = E[N(x)] = \sum_{n=1}^{\infty} F_n(x) < \infty.$$

The function H(x) is called the renewal function and it gives the mean number of renewals occurring in the interval (0,x). The probability of having exactly n renewals of an event in the intervals [o, t] given as

$P[\mathbf{N}(t) = n] = F_n(t) - F_{n+1}(t)$ has many real life applications.

Sums of dependent random variables

Gurland (1955) has shown that the characteristic function $\phi(\lambda_1, \lambda_2 \ldots \lambda_n)$ of the joint distribution of any n random variables from a sequence $\{X_n\}$ of exchangeable random variables each following the exponential distribution with p.d.f. $f(x) = \left(\frac{1}{a}\right) e^{-x/a}, a > 0, 0 < x < \infty$ such that the correlation coefficient R between any X_i and X_j, $i \neq j$ (independent of i and j) is given by

$$\phi(\lambda_1, \lambda_2 \cdots\cdots \lambda_n) = \begin{vmatrix} 1 - i\lambda_1 a & - & i\lambda_1 aR & \cdots & -i\lambda_1 aR \\ -i\lambda_2 aR & & 1 - i\lambda_2 a & \cdots & -i\lambda_2 aR \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -i\lambda_n aR & & -i\lambda_n aR & & 1 - i\lambda_n a \end{vmatrix}^{-1}$$

(A sequence $\{X_n\}, n = 1, 2, \cdots,$ random variables is called a sequence of exchangeable random variables, if the joint distribution $F_n(x_1, x_2, \ldots . x_n)$ of any n variables can be expressed as

$F_n(x_1,x_2,\ldots.x_n) \quad = \int_\Omega G_W(x_1),\cdots G_W(x_n)dp(w),\ n=1,2,\cdots,$

where $G_w(x)$ is a conditional distribution function in x for each w and a random variable in w for a given x. Here Ω is the space of w).

The cdf of $S_n = x_1 + x_2 + \ldots.+ x_n$ is

$F_n(x) = Pr[S_n \le x]$

$$= (1-R)\sum_{i=0}^{\infty} \frac{(nR)^i}{(1-R+nR)^{i+1}} \frac{\psi(n+i, x/b)}{(n+i-1)!}$$

where $\psi(n,x) = \int_0^x e^{-\tau}\tau^{n-1}d\tau$ and $b = a(1-R)$

It has been proved that the Laplace Stieltjes transform of $F_n(t)$ is given by

$$F_n^*(s) = s\int_0^\infty e^{-st}(1-R)\left[\sum_{i=0}^{\infty} \frac{(nR)^i}{(1-R+nR)^{+1}} \frac{\psi(n+i, x/b)}{(n+i-1)!}\right]dt$$

$$= \frac{1}{(1+bs)^k} \frac{1}{\left[1+\frac{1+nR}{(1-R)}\frac{(bs)}{(1+bs)}\right]}$$

This result is used in the succeeding chapters to obtain the Laplace Stieltjes transform of L(t), which in turn is used for the evaluation of E(T) and V(T).

In this dissertation Chapter II contains a brief of survey of results in Manpower Planning especially related to the survey of wastage process.

In Chapter III a Stochastic Model for estimation of the expected time to recruitment and its variance and derived assuming the inter arrival times between decision epochs form order statistics.

In Chapter IV Stochastic Model for the estimation of E(T) and variance are derived under the assumption that the random amount of wastage on successive decision epochs are constantly correlated.

In Chapter V, a Stochastic Models for estimation of E(T) and its variance are derived under the assumption that at each decision making epoch, some preventive strategies adopted, so as to avoid wastage or atleast some reduction in the amount of wastage.

In Chapter VI a different kind of a model is derived. In developing the stochastic model, the use of a special kind of a process and to find E(T) and variance under the assumption that the inter arrival times between successive decisions epochs form a Geometric process.

Chapter VII devoted to summary of results discussed in this thesis and also the conclusions that could be drawn. Also some suggestions are indicated for future work in this direction.

Chapter – II

Review of Some Results in Manpower Planning

CHAPTER – II

REVIEW OF SOME RESULTS IN MANPOWER PLANNING

In the area of study relating to manpower planning a number of research papers have been published since 1970. The main topics of discussion are pertaining to the following aspects of study.

1. Recruitment
2. The transitions or flows between grades in a hierarchical system.
3. Wastage process
4. Predictions regarding staff strength and recruitments
5. Optimization methods for finding optimal staff strength which would be cost effective.

A brief summary of such research papers by different authors is furnished in this chapter.

The prediction of the number and distribution of staff at a future time point in various grades of an organization is extremely useful for making policy decisions regarding recruitments. Young and Almond (1961) have discussed a mathematical model which can be used for the purpose of the above. In deriving the model the authors have considered the concepts like pattern of recruitment, promotion and wastage. The model comprises the mathematical equations involving the above components.

A general description of the models and the methods for developing mathematical models for manpower planning studies have been discussed in a broadway by Dill et al.(1966). In this paper the authors have explored some results in manpower and the development of some simple stochastic models to take note of such issues in manpower planning. Direct mathematical methods have been used for structuring such models and the simulation methods have been used to a large extent in such models.

Bartholomew (1971) has given a full fledged discussion about the statistical approach to manpower, planning. A conceived description of manpower system, the various components of manpower system and a brief account of the historical development of this discipline of science are all provided in this paper. The author has given a vivid description of labour wastage, the various measures of wastage and the impact of the Completed Length of Service (CLS) distributions on the wastage phenomenon. A brief account of stocks and flows in any organization is also defined. A Markov model and prediction equations for stocks and flows in a graded organization are all discussed in this paper.

The Cox regression model (1972) is used to find the propensity of leaving a job in which both CLS and personal factors are taken into account for study where CLS is described by a quantitative variable while personal factors are described by qualitative variables.

Marshall(1973) has discussed two mathematical models for personnel movement in a hierarchical organization. The first model is a Markov model and the second model is a cohort model. The comparison of two mathematical models is also given in this paper. $X_i(t)$ and $X_j(t)$ are defined as the number in grade i and j respectively at time t. The author defines a matrix Q(t)

$$Q(t) = [q_{ij}(t)]$$

Where $q_{ij}(t)$ is the fraction of those in grade i at time t who will move to state j at time t + 1. The expected number in each grade at time t + 1 given the row vector $X(t) = [X_1(t), \ldots X_k(t)]$ is given by the expression

$$E_n[X\ (t)] = [X(t)\ Q(t) + Y(t + 1)]$$

This expression for the expected number in each grade at a future time point is known us the Markov Chain model. Another model, called Cohort model is also derived for the purpose of giving the expected number of persons in each grade. The author uses two covariance matrices C(t) and B(t). The structure of these two matrices is also given in this paper. Numerical illustration using the number of students in each class, the forecast of enrollment is also discussed in this paper.

Bartholomew (1973) has discussed a hierarchical manpower system in which transitions between grades are permitted. The viability of transitions between grades enables the possibility of transfer of personnel between grades. Such a strategy would help compensating loss of manpower at grades where larger wastage is reported.

Young and Vassiliou (1974) have considered a model relating to promotion and the model is of non-linear character. The organization is interpreted as a graded social system with different states and the transition between states occur due to promotion, wastage and recruitment. The reason for comparing the organization as a graded social system is due to the reason that in a society the transition may be observed from one stratum to the other due to various reasons and they may be classified as

a) Ecological principles which state that the promotions are according to the size of staff available in a given grade.

b) The principle of fairness which depends upon the perceived profile of the individuals and the judgment is made by the suitable evaluation of the individual's capability that makes fair comparisons with previous evaluations.

Vajda (1975) has given a very systematic account of the applications of mathematical models to the problems of manpower. In any organization the employees can be partitioned into different grades. So, a population of employees in an organization has a well defined structure in terms of the partition. The question is to find out the changes in a given structure and how it gives rise to another structure after one or more transitions. The second question is from a given structure in how many steps a prespecified structure could be attained. The problem of interest here is to find out in how many transition steps the structure which is identical with the starting could be attained. It implies the re-visit to the starting state in the terminology of stochastic processes.

Many mathematical models in manpower planning are based on optimization techniques. The optimization may be with regard to the utilization of available manpower, the optimal policies regarding recruitment optimal duration of training etc. A number of models have been developed in this direction during the last three decades. These models invariably use the different methods of mathematical programming. Schneider and Kilpatric (1975) proposed an interesting model relating to optimal manpower modeling where the authors have taken up the optimum utilization of medical manpower available in relation to the demand for the same. The interaction between effective manpower utilization, faculty requirements and available characters are also considered and two basic models are given relating to minimization of medical manpower needs and the maximization of subscribers.

A mathematical model of a military manpower system with a view to determine the optimal steady state, wage rate and force distribution by length of service is by Jaquette and Nelson (1976). In this paper it is assumed that the cost of hiring personnel is determined by military manpower supply function which relates enlistment and re-enlistment rates to military pay. The optimal force is defined as that force which provides the greatest military capability for a given budget cost. Optimal rates of pay are determined by maximizing the productivity index subject to a budget constraint. Assuming the basic flow process as Markovian the optimal rates of pay are determined. The steady state optimal policy for the Cobb-Douglas type function is obtained using the Lagrangian multiplier technique. Numerical results are also discussed.

The use of linear programming methods to derive optimal long term policy has been discussed by Grinold and Marshall (1977). They introduce the long term planning horizon and uncertain conditions pertaining to future manpower requirements. The input data are regarding future requirements, budget constraints, costs, discount rate, utilization factor and coefficient factor relating to flow processes. The inflow is taken as decision variable, the objective function being the discount cost. The minimization of the same is discussed.

Lesson (1979) has considered a model in which a graded manpower system is specified and the wastage rate, promotion rate, length of service of employees in each grade are all considered. Assuming these rates the author finds out the transition probabilities between the different grades and also examines the stationarity of transition probabilities.

Mukherjee and Chattopadhyay (1985) have discussed an optimal recruitment policy. The authors have considered an organization in which numbers of persons are recruited at time t. Each recruitee can be in service for a period of τ years at the most. It is also assumed that the efficiency of each recruited person is adversely affected by the longer duration of service. The authors have derived a recruitment policy at interval of time t. The optimal value of t which minimizes the total cost of unfilled vacancies and forced retirement have been worked out.

Goh et al.(1987) have discussed an optimal policy which could be used for manpower training programmes. Two models are discussed taking a finite planning horizon and an infinite planning horizon. The authors considered an organization which has a total of N employees. Training programmes are considered for a fixed period with a view to increase the efficiency of workers. The returns arising out of training a programme is also assessed. The authors have attempted in evolving training policies which would maximize the expected total return over the entire total planning horizon. It is also suggested that the number and extent of training programmes which would involve minimum cost can also be found alternatively.

A dynamic programming approach to determine optimal manpower recruitment policies has been discussed by Poornachandra Rao (1990). The various manpower system costs such as recruitment cost, overstaffing cost, understaffing cost, firing/retirement cost, retenion cost are all outlined in this model. The recruitment cost in period t is taken to be the concave function $S_t\, \delta(x_t) + \upsilon x_t$ where

$$\delta_{xt} \begin{cases} = 0 & \text{if } x_t = 0 \\ = 1 & \text{if } x_t > 0 \end{cases}$$

where x_t is the number of persons recruited in period t. The overstaffing cost is denoted as $i_t I_t$, where i_t, is the cost of overstaffing per recruitee per period and I_t is the number of persons recruited in an earlier period for the requirement of period t.

The total cost of recruitment for T period of planning interval is given by

$$\sum_{t=1}^{T} \left[S_t \delta \left(x_t\right) + V\, x_t + i_t I_t\right]$$

Here V stands for the variable cost of recruitment per employee recruited. So, the problem is stated as

$$\text{Minimize} \sum_{t=1}^{T} \left[S_t \delta + i_t I_t\right]$$

$$\text{Subject to} \sum_{k=1}^{t} x_k = \sum_{k=1}^{t} R_k, t = 1, 2, \cdots, T$$

The author has also given a dynamic programming formulation of this model and given the algorithm to solve the problem. Numerical illustration is also discussed.

This non-linear model is used to predict the staff movements and it involves the solution to the differential difference equation.

Biswas and Adhikari (1992) have considered both quantitative and qualitative characters for studying manpower wastage and attrition.

An analytical model which deals with finding the size of each grade in a hierarchical organization has been developed by Kennay et al. (1997). The size of each grade and proportion of employees in each grade are derived using a number of assumptions regarding the demand for manpower in that organization, the wastage, rate the recruitment, etc., The concept of promotion is also taken into account. It is assumed that the demand for manpower increases exponentially at a rate 'ρ'. The wastage rate $w(x, t)$ for employees of age x at time 't' is defined as the proportion $w(x, t)\ \delta t$ of staff at age x at time t who leave the organization $(t, t + \delta t)$. $N(t)$ is the total size of the system at time t, $P(t)$ is taken to be the proportion of the employees in the top grade, $R(t)\ \delta t$ denotes the total number of recruits in an interval $(t, t + \delta t)$. A constraint taken up is

$P(t) = \int_0^t f(x,t)\, g(x,t)\, dt$ where f(x, t) is defined as the proportion of all employees age x at time 't' who have been promoted to the top grade and g(x, t) is the age distribution of all employees at time t. The constraint that demotion is not permitted is represented as $f(x + \alpha, t + \alpha) \geq f(x, t)$ for all x, t, α. Using all the above the authors have obtained a model for the sizes of the different grades at a future point of time 't'.

Subramanian (1997) has proposed a manpower model in which the optimal training duration for personnel in an organization is determined. In doing so it is assumed that the technical personnel in an organization are given training towards updating their knowledge. Such trained persons may leave the organization and in such cases the vacancies are filled by qualified personnel by direct recruitment. In the first model, the optimal training duration T is derived assuming that only one person is deputed for training at a time point. In the second model, all the persons are recruited initially are given training at each of 'r' epochs of training. In this case the optimal interval between training programmes is obtained.

Shock model approach to determine the expected time for the recruitment has been discussed by Sathiyamoorthi and Elangovan (1998). In this paper the authors have assumed that the organization announces policy revisions regarding sales target, wage revisions, etc., Whenever such policy revisions are announced there is wastage of personnel to a random extent. As and when the total wastage crosses a random thereshold level the organization is rather compelled to take up recruitments. Assuming different thereshold distributions, the authors have obtained numerical results for the expected time to recruitment.

Sathiyamoorthi and Elangovan (1999) have discussed the determination of the optimal time interval between recruitment programmes under certain specific assumptions of cost. It is assumed that the persons are recruited and trained. Trained persons are absorbed into the organization as and when vacancy arises. The cost of training is assumed to be very high and hence, it is recovered at a higher rate from the trained employed persons, and at a lower rate from those who are trained but not employed. The optimal time interval between training programmes which is T is obtained. In another model, assuming that the shortage of trained personnel gives rise to loss of productivity, the optimal T is obtained.

Sathiyamoorthi and Parthasarathy (2002) have considered a two grade organization in which the mobility of personnel from one grade to the other is permitted so as to compensate the loss of manpower which is larger among the two grades. They have considered the case in which the Max (Y_1, Y_2) is taken to be the threshold level of the organization where Y_1 and Y_2 are the individual thresholds of the grades respectively.

Sathiyamoorthi and Parthasarathy (2003) have used the idea of change of parameter for the threshold distribution after the truncation point. The idea is similar to SCBZ property where the parameter undergoes a change. Assuming the truncation point is itself a random variable following exponential distribution, the authors derived a distribution function, which is taken for the threshold level. The expected time for recruitment is also obtained using the shock model approach and the results are compared when there is no change of parameter for the threshold distribution.

Chapter – III

A Stochastic Model based on Order Statistics for Estimation of Expected Time to Recruitment

CHAPTER – III

A STOCHASTIC MODEL BASED ON ORDER STATISTICS FOR ESTIMATION OF EXPECTED TIME TO RECRUITMENT

3.1 Introduction

In Business Organizations decisions regarding the revision of pay and other packages together with target of achievements are taken at random time intervals. Consequent to such a decision the depletion which is also called the wastage, is imminent due to the fact that the packages may not be statisfactory or targets may be high or both. This is very common in marketing organizations. The impact of the length of interarrival times irrespective of the amount of wastage on each decision epoch is studied in this chapter. In doing so the concept of order statistics is used. The expected time to recruitment and its variance are obtained using the shock model and cumulative damage process approach.

The effect of increasing in the length of inter arrival times between decisions on the values of E(T) and V(T), has been found out.

It may be noted that in any organization or industry, the leaving of personnel is a common phenomenon. There are two different kinds of leaving processes. The first is involuntary, where a person leaves the organization due to retirement, deaths etc. It is not under the control and perview of the individual concerned. On the other hand the voluntary type of leaving process is one in which decision to leave the organization is made by the individual concerned. It may be voluntary retirement, switching over to other organizations etc. This kind of a decision may be due to the push factors and pull factors.

In business organizations, industries etc., it is usual that policy decisions regarding pay revisions, perquisites, incentives and also targets are announced periodically. This is very common in marketing organizations. At every such epoch it is quite common that some persons may leave the organization which leads to wastage or depletion of manpower. The organization cannot immediately resort to recruitments on each occasion, since recruitment and subsequent training involves associated costs. For a detailed study of recruitment cost one can refer to Poornachandra Rao (1990).

The policy decisions are taken mostly at random time intervals. Hence the interarrivaltimes between policy decision epochs are assumed to be identically, independently distributed (i.i.d) random variables. The organization attempts to sustain the activities intact, inspite of the wastage on each decision epoch. The wastages are cumulative. As and when the total wastage crosses a particular level is called the threshold, the activities can no more be sustained, in other words it is a breakdown point which results in immediate recruitment of personnel.

Sathiyamoorthi and Elangovan (1999) have discussed a model for the method of finding the expected time to recruitment using shock model and cumulative damage process. (CDP). For a detailed study of shock models and CDP one can refer to Esary, Proschan and Marshall (1973).

In this chapter the concept of order statistics is used to represent the distribution of interarrival times between successive decision making epochs. Since the interarrival times between decision epochs are assumed to be i.i.d., random variables, they can be arranged as a monotonic increasing sequence, there by giving rise to order statistics. The expected time to recruitment and its variance are obtained taking the distribution of interarrival times as first order (minimum) and n^{th} order (maximum) statistics respectively. The relative changes in the expected time to recruitment are obtained. It helps to assess the impact of the interarrival times (or) periods between decision epochs on the expected time to recruitment.

3.2 Assumptions of the model:

(i) *The random amounts of manpower wastage on* the k decision epochs are X_1, $X_2 \ldots X_k$ and represented in terms of man-hours.

(ii) The amount of manpower wastage X_i, i = 1,2, ...,k and the threshold Y are statistically independent.

(iii) The wastage of manpower on successive decision epochs are linear and cumulative.

3.3 NOTATION:

X_i : A Continuous random variable denoting the amount of depleteion in man hours due to i^{th} decision. i = 1,2, ...,k with p.d.f g(.) and c.d.f. G(.)

Y : A Continuous random variable denoting the threshold level (or) breakdown point and has p.d.f. h(.) with c.d.f. H(.)

U_i : Continuous random variable denoting the interarrival times between decision epochs. i = 1,2, ...,k and with p.d.f f(.) and c.d.f. F(.)

$U_{(1)}$: The smallest order statistic and has p.d.f $f_{(1)}(.)$

$U_{(k)}$: The largest order statistic and has p.d.f $f_{(k)}(.)$.

f*(.): Laplace transform of f(.).

T : A Continuous random variable denoting the time for the cumulative loss to cross the threshold level (i.e)., time to breakdown of the system with p.d.f $l(.)$ and c.d.f. L(.)

$G_k(.)$: p.d.f of sum $X_1 + X_2' \ldots X_k$

$F_k(.)$: K convolution of F(.)

3.4 RESULTS

If X_i, i = 1,2, ...,k are random variables then $\sum_{i=1}^{k} X_i$ is also a random variable and it can be shown that

$$P\left[\sum_{i=1}^{k} X_i < y\right] = \int_0^{\infty} g_k(x)[1 - H(x)]dx$$

If we assume that y ~ exp (θ) then

$$P\left[\sum_{i=1}^{k} X_i < y\right] = \int_0^{\infty} g_k(x) e^{-\theta x} dx$$

$g_k^*(\theta) = [g^*(\theta)]^k$ since, X_i's are i.i.d., ...(3.1)

If X_i, i = 1, 2, ...,k are the random amounts of manpower wastage (in man hours), linear and cumulative, with threshold Y then the survivor function is given by S(t) = P [T > t]

Pr [the system does not reach the breakdown state before t]

$$= \sum_{k=0}^{\infty} \Pr \text{[exactly 'k' decision epochs in (0,t)] x}$$

P [The cumulative man power depletion < y].

Now the probability that there are exactly k decision epochs in (0, t) is given by $\sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right]$ by renewal theory.

Hence $P[T > t] = \sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right] \Pr\left[\sum_{i=1}^{k} X_i < y\right]$

$$= \sum_{k=1}^{\infty} \left[F_k(t) - F_{k+1}(t)\right] \; [g^*(\theta)]^k \text{ from (3.1)}$$

$$= 1 - [1 - g^*(\theta)] \sum_{k-1}^{\infty} F_k(t) \; [g^*(\theta)]^{k-1} \text{-on simplification} \quad \ldots(3.2)$$

Hence

P[T < t] = L(t) = 1 – S(t)

$$= [1 - g^*(\theta)] \; \sum_{k-1}^{\infty} F_k(t) \; [g^*(\theta)]^{k-1}. \qquad \ldots (3.3)$$

Taking Laplace transform of (3.3) both sides,

$$L^*(s) = [1 - g^*(\theta)] \; \sum_{k-1}^{\infty} F_k^*(s) \; [g^*(\theta)]^{k-1} \qquad \ldots(3.4)$$

Now $l^*(s)$ is the Laplace transform of the p.d.f $l(t)$

and $f^*(s)$ is the Laplace transform of f(.) and $L^*(s) = \frac{l^*(s)}{s}$

$$F^*(s) = \frac{f^*(s)}{s}$$

From (3.4) it can proved that

$$l^*(s) = [1 - g^*(\theta)] \; \sum_{k-1}^{\infty} \left[f^*(s)\right]^k \; [g^*(\theta)]^{k-1}$$

$$= [1 - g^*(\theta)] \; (f^*(s)) \sum_{k=1}^{\infty} [f^*(s) g^*(\theta)]^{k-1}$$

$$= \frac{[1 - g^*(\theta)] \; f^*(s)}{[1 - g^*(\theta) \; f^x(s)]} \qquad \text{on simplification} \qquad \ldots(3.5)$$

To find the E(T), the expected time to break down / recruitment, we have

$$E(T) = \left. \frac{-dl^*(s)}{ds} \right|_{s=0}$$

$$E\left(T^2\right) = \left. \frac{d^2 l^*(s)}{ds^2} \right|_{s=0}$$

From which V(T) can be obtained. This is the result in the most general form of the inter arrival times of decision epochs where U_i's i = 1,2, ...,k are i.i.d.

Let U_1, U_2, ...U_k be arranged in increasing order of magnititude so that we have the sequence as $U_{(1)}$, $U_{(2)}$, ...$U_{(k)}$. Here r = 1,2,...,k are called the order statistics and with $U_{(1)}$ as the first order statistic (smallest) and $U_{(k)}$ the k^{th} order statistic (largest) and the random variables $U_{(1)}$, $U_{(2)}$, ...$U_{(k)}$ are not independent.

The p.d.f of the 'r'th order statistic is given by

$$f_{u(r)}(t) = r\binom{n}{r} [F(t)]^{r-1} f(t)[1-F(t)]^{n-r} \qquad \ldots(3.6)$$

If we take r = 1 we get the p.d.f of the first order statistics $U_{(1)}$ as,

$$f_{u(1)}^{(t)} = k\,[1-F(t)]^{(k-1)} f(t) \qquad \ldots(3.7)$$

In a similar way if we take r = k, the p.d.f of the k[th] order statistic is given by

$$f_{u(k)}^{(t)} = k\,[F(t)]^{k-1} f(t) \qquad \ldots(3.8)$$

Case i

Now let $f_{u_{(1)}}^{*}(s)$ be the Laplace transform of $f_{u_{(1)}}(t)$,

Then substituting this in (3.5), the expression for $l^*(s)$ can be obtained as

$$l^*(s) = \frac{[1-g^*(\theta)]\; f_{u_{(1)}}^{*}(s)}{\left[1-g^*(\theta)\; f_{u_{(1)}}^{*}(s)\right]} \qquad \ldots(3.9)$$

If it assumed that f(t) is distributed as exponential with parameter λ, then the Laplace transform of $f_{u_{(1)}}^{*}(t)$ is given by

$$f_{u_{(1)}}^{*}(s) = \frac{k\lambda}{k\lambda + s} \qquad \ldots(3.10)$$

Substituting (3.10) in (3.9) we get

$$l^*(s) = \frac{[1 - g^*(\theta)]\dfrac{k\lambda}{k\lambda + s}}{\left[1 - g^*(\theta)\dfrac{k\lambda}{k\lambda + s}\right]} \qquad \ldots(3.11)$$

Special case

If it is assumed that g(.) follows exponential distribution with parameter 'c' then $g^*(\theta) = \frac{c}{c+\theta}$.

Substituting this in (3.11) we get

$$l^*(s) = \frac{\left[1 - c/_{c+\theta}\right]\dfrac{k\lambda}{k\lambda + s}}{\left[1 - c/_{c+\theta}\dfrac{k\lambda}{k\lambda + s}\right]}$$

$$= \frac{k\lambda\theta}{[\theta\lambda k + (c+\theta)s]} \quad \text{on simplification} \qquad \ldots(3.12)$$

Now to find E(T) we take

$$E(T) = \left.\frac{-dl^*(s)}{ds}\right|_{s=0}$$

$$= \frac{(C+\theta)\,\theta k\lambda}{(k\lambda\,\theta)^2}$$

$$= \left(\frac{c+\theta}{k\lambda\,\theta}\right) \qquad \ldots(3.13)$$

Similarly it can be shown that

$$E\left(T^{2}\right)=\left.\frac{-d^{2}l^{*}(s)}{ds^{2}}\right|_{s=0}$$

$$=\frac{2(\theta k\lambda)\ \ (C+\theta)^{2}}{(k\lambda\ \ \theta)^{3}}$$

$$=\frac{2(C+\theta)^{2}}{(k\lambda\ \ \theta)^{2}}$$

$$=2\left(\frac{c+\theta}{k\lambda\ \theta}\right)^{2} \qquad \ldots(3.14)$$

and

$V(T) = E(T^2) - [E(T)]^2$

$$=2\frac{(c+\theta)^{2}}{(k\lambda\ \theta)^{2}}-\left(\frac{c+\theta}{k\lambda\ \theta}\right)^{2}$$

$$=\left(\frac{c+\theta}{\theta\lambda k}\right)^{2} \text{ on simplification.} \qquad \ldots.(3.15)$$

Case ii

Let us take the p.d.f of the k^{th} order statistic $U_{(k)}$.

$$f_{u(k)}^{(t)}=k\,[F(t)]^{k-1}\,f(t)$$

Again assuming that f(t) follows exponential distribution with parameter λ, we have

$$f_{u(k)}^{(t)}=k\left[1-e^{-\lambda t}\right]^{k-1}\lambda.e^{-\lambda t}$$

$$= k\lambda e^{-\lambda t}\left[1 - e^{-\lambda t}\right]^{k-1}$$

Now $f^{*}_{u(k)}(s) = k\,\lambda \int_{0}^{\infty} e^{-st} e^{-\lambda t}\left(1 - e^{-st}\right)^{k-1} dt.$

If can be shown that

$$f^{*}_{u(k)}(s) = \frac{k!\lambda^{k}}{(\lambda + s)(2\lambda + s)\cdots(k\lambda + s)} \text{ on simplification} \quad \ldots(3.16)$$

Now assuming as before that g(.) is exponential with parameter 'c' we have $g^{*}(\theta) = \frac{c}{c + \theta}$

We have,

$$l^{*}(s) = \frac{\theta k!\lambda^{k}}{(c + \theta)(\lambda + s)(2\lambda + s)\cdots(k\lambda + s) - c.k!\lambda^{k}} \quad \ldots.\ (3.17)$$

Now

$$E(T) = \left.\frac{-dl^{*}(s)}{ds}\right|_{s=0}$$

We have, Now

$$E(T) = \frac{-d}{ds}\left[\frac{k!\,\lambda^{k},\theta}{(c + \theta)A - ck!\,\lambda^{k}}\right]_{s=0}$$

Where A = $(\lambda + s)\ (2\lambda + s)\ldots\ldots.\ (k\lambda + s)$

Now $\left.\frac{dA}{ds}\right|_{s=0} = 1\ (2\lambda)\ (3\lambda)\ \cdots\ (k\lambda)$

$$+\lambda\,(3\lambda)\cdots(k\lambda).\ \times\frac{1}{2}$$

$$+\lambda\,(2\lambda)\frac{3}{3}\cdots(k\lambda).\ \times\frac{1}{3}$$

....

....

$$=\lambda 2\lambda\quad 3\lambda\quad\cdots\quad k\lambda\times\frac{1}{\lambda}$$

$$=\lambda^{k-1}\,k!\,\left\{1+\frac{1}{2}+\frac{1}{3}+\cdots\right\}$$

$$=\lambda^{k-1}\,k!\,\sum_{n=1}^{k}\frac{1}{n}$$

$$E(T)=\frac{k!\,\lambda^{k},\theta\,(c+\theta)}{\left[(c+\theta)A-ck!\,\lambda^{k}\right]^{2}}\cdot\frac{dA}{ds}\bigg|_{s=0}$$

$$=\frac{k!\,\lambda^{k}\theta\,(c+\theta)}{\left[(c+\theta)A-ck!\,\lambda^{k}\right]^{2}}.\lambda^{k-1}k!\,\sum_{n=1}^{k}\frac{1}{n}$$

$$=\frac{k!\lambda^{k}\,\theta(c+\theta)}{\left[(c+\theta)k!\lambda^{k}-c.\,k!\lambda^{k}\right]^{2}}\lambda^{k-1}\,k!\sum_{n=1}^{k}\frac{1}{n}$$

$$=\frac{k!\lambda^{k}\,\theta(c+\theta)\,k!\,\lambda^{k-1}}{\left(k!\,\lambda^{k}\right)^{2}\quad\left[(c+\theta)-c\right]^{2}}\sum_{n=1}^{k}\frac{1}{n}$$

$$=\frac{c+\theta}{\lambda\,\theta}\left(\sum_{n=1}^{k}\frac{1}{n}\right)$$

Using this in the expression for E(T) we get.

$$E(T)=\left(\frac{c+\theta}{\theta\lambda}\right)\left(\sum_{n=1}^{k}1/n\right) \qquad \dots (3.18)$$

It implies that E(T) increases as 'k' increases. This other words means that as the length inter arrival times between wastages increases then the expected time to recruitment also increases.

Now

$$\ell^*(s)=\frac{\theta\, k!\lambda^k}{(c+\theta)A-ck!\lambda^k}$$

$$\frac{d\ell^*(s)}{ds}=\frac{(-1)\theta\, k!\lambda^k}{\left[(c+\theta)A-ck!\lambda^k\right]^2}(c+\theta)\frac{dA}{ds}$$

$$\frac{d^2\ell^*(s)}{ds}=\frac{\left(\frac{dA}{ds}\right)(c+\theta)(2)\theta\, k!\lambda^k}{\left[(c+\theta)A-ck!\lambda^k\right]^3}(c+\theta)\frac{dA}{ds}-\frac{-\theta\, k!\lambda^k}{\left[(c+\theta)A-ck!\lambda^k\right]^2}(c+\theta)\frac{d^2A}{ds^2}$$

$$\frac{2(c+\theta)^2\ \theta\, k!\lambda^k}{\left[(c+\theta)A-ck!\lambda^k\right]^3}\left(\frac{dA}{ds}\right)^2-\frac{\theta\, k!\lambda^k(c+\theta)}{\left[(c+\theta)A-ck!\lambda^k\right]^2}\frac{d^2A}{ds^2}$$

$$E\left(T^2\right)=\frac{2k!\,\theta\lambda^k(c+\theta)^2}{\left[(c+\theta)A-ck!\,\lambda^k\right]^3}\left(\frac{dA}{ds}\right)^2 - \left.\frac{k!\,\theta\,\lambda^k(c+\theta)}{\left[(c+\theta)A-ck!\,\lambda^k\right]^2}\frac{d^2A}{ds^2}\right|_{s=0}$$

Now it can be verified that

$$\left.\frac{d^2A}{ds^2}\right|_{s=0} = k!\ \lambda^{k-2}\left[\sum_{n=1}^{k}\left(\frac{1}{n}\right)^2 - \sum_{n=1}^{k}\frac{1}{n^2}\right]$$

Hence

$$E\left(T^2\right) = \frac{2(c+\theta)^2}{\theta^2\lambda^2}\left(\sum_{n=1}^{k}\frac{1}{n}\right)^2 - \frac{(c+\theta)}{\lambda^2\theta}\left(\sum_{n=1}^{k}\frac{1}{n}\right)^2$$

$$+\frac{(c+\theta)}{\lambda^2\theta}\sum_{n=1}^{k}\frac{1}{n^2}$$

From the above it can be shown that

$$V(T) = \frac{C(C+\theta)}{\theta^2\lambda^2}\left[\sum_{n=1}^{k}\frac{1}{n}\right]^2 + \frac{C+\theta}{\lambda^2\theta}\sum_{n=1}^{k}\frac{1}{n^2} > 0 \qquad \text{... (3.19)}$$

The variance of T also increases as 'k' increases (i.e) if the length of interarrival times between wastages increases then the time to recruitment also shows greater variations.

The above results can be represented graphically by taking suitable numerical illustration.

Numerical Illustration

Case (i)

For the first order statistic $U_{(1)}$.

We know that

$$E(T) = \frac{(c+\theta)}{k\lambda\,\theta}$$

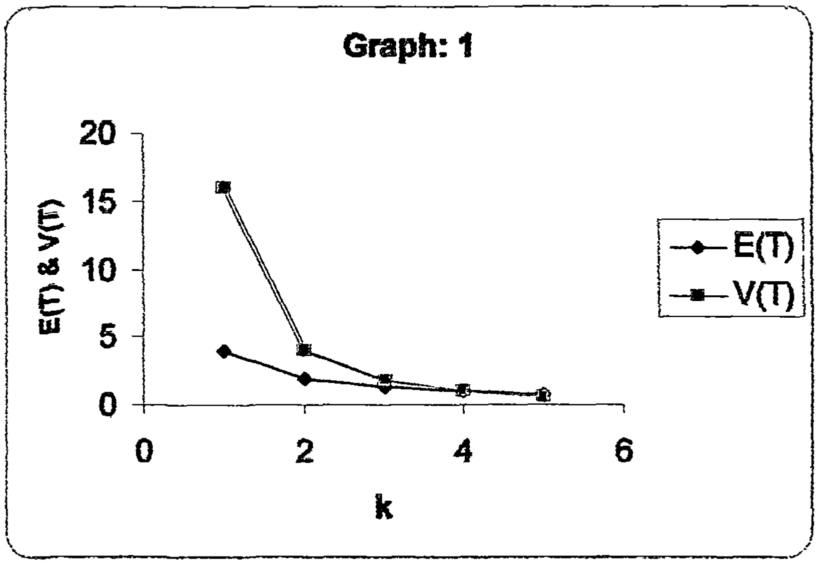
$$V(T) = \left(\frac{c+\theta}{\theta\lambda k}\right)^2$$

Now k = number of decision epochs. We find out the values of E(T) and V(T) for k = 1,2,......10. for fixed values of the parameters, and θ, c, λ.

Table: 1

$\theta = 0.5$, $c = 1.5$, $\lambda = 1.0$.

k	E(T)	V(T)
1	4.0	16.0
2	2.0	4.0
3	1.3	1.8
4	1.0	1.0
5	0.8	0.6
6	0.7	0.4
7	0.7	0.4
8	0.5	0.3
9	0.4	0.2
10	0.4	0.2



It is observed that if 'k' which is the number of decision epochs in (0, t) increases, then the length of interarrival times is shorter. Hence E(T) and V(T) both become smaller, as the decision epochs are more all more. This is indicated in graph 1.

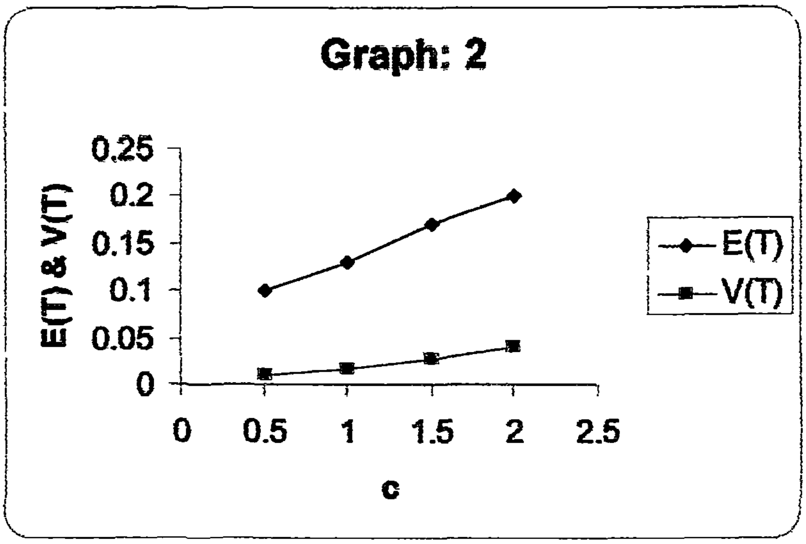
Case (ii)

In order to study the impact of variations in the parameter 'c' on the values of E(T) and V(T). We find E(T) and V(T) in table 2.

Table - 2

$k = 10 \quad \theta = 1.0, \quad \lambda = 1.5.$

c	E(T)	V(T)
0.5	0.1	0.01
1.0	0.13	0.0178
1.5	0.17	0.0278
2.0	0.2	0.04



It can be seen from the above that as 'c' the parameter of the exponential distribution of the random variable X_i, which indicates the amount of wastage on the decision epochs increases, then the E(T) increases. This is due to the fact that $E(x) = \frac{1}{c}$ since X has exponential distribution with parameter c. So as C increases $\frac{1}{c}$ decreases, (i.e.,) the amount of wastage on each decision epoch is on the average smaller. Hence it would take longer time tot cross the threshold level of wastage. So E(T) increases, accordingly.

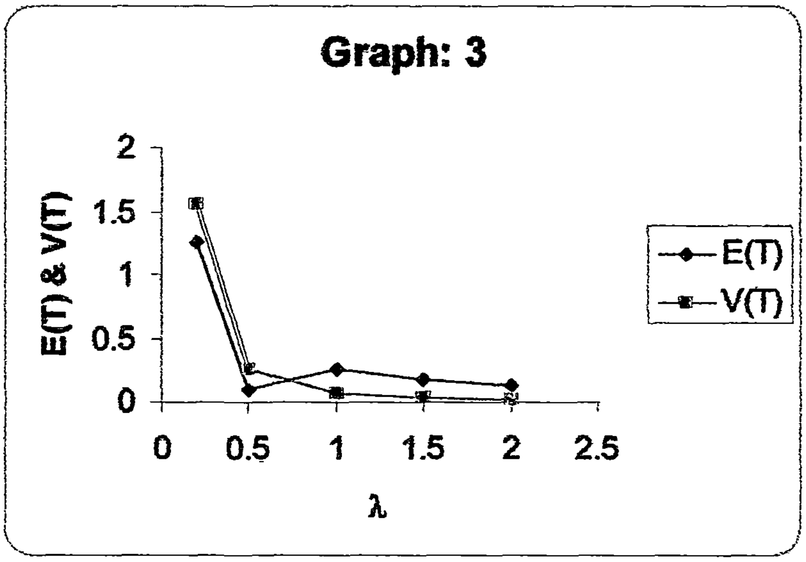
Case (iii)

k, θ, and c are fixed. λ is allowed to vary.

Table: 3

k = 10 θ = 1.0, C = 1.5.

λ	E(T)	V(T)
0.2	1.25	1.5625
0.5	0.1	0.25
1.0	0.25	0.0625
1.5	0.17	0.028
2.0	0.125	0.016



As λ increases $E(T) = \frac{1}{\lambda}$ decreases. Hence the average interarrival time decreases. So the E(T) also decreases.

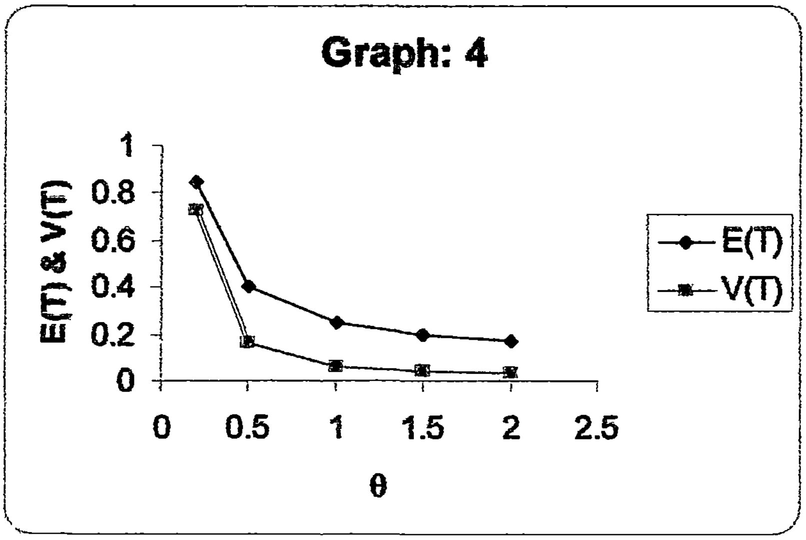
Case (iv)

Table: 4

k, λ and C and fixed. θ is allowed to vary.

k = 10   λ = 1.0,   C = 1.5.

θ	E(T)	V(T)
0.2	0.85	0.7225
0.5	0.4	0.16
1.0	0.25	0.0625
1.5	0.2	0.04
2.0	0.175	0.031



If the parameter θ of the threshold distribution increases, then E(Y) = $\frac{1}{\theta}$ implies that the average values of Y decreases. So, the threshold is decreasing in nature and so E(T) also decreases.

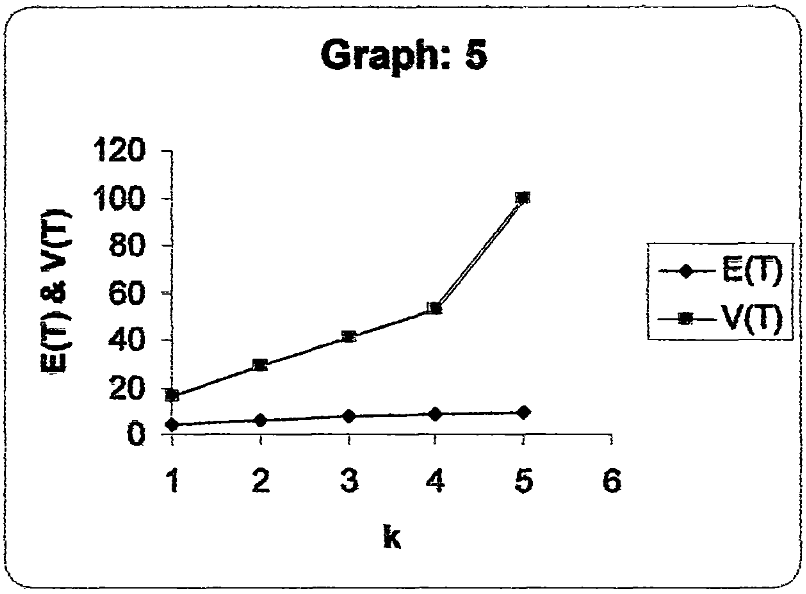
Case (v)

For the k^{th} order statistics $U_{(k)}$.

Table: 5

We fix $\theta = 0.5$, $c = 1.5$, $\lambda = 1.0$, K is allowed to vary.

k	E(T)	V(T)
1	4.0	16.0
2	6.0	28.78
3	7.32	41.09
4	8.32	52.88
5	9.12	99.99



It may be noted that as K increases the K^{th} order statistics also increases, which implies that the interarrival times between wastages increases. Hence E(T) shows an increase.

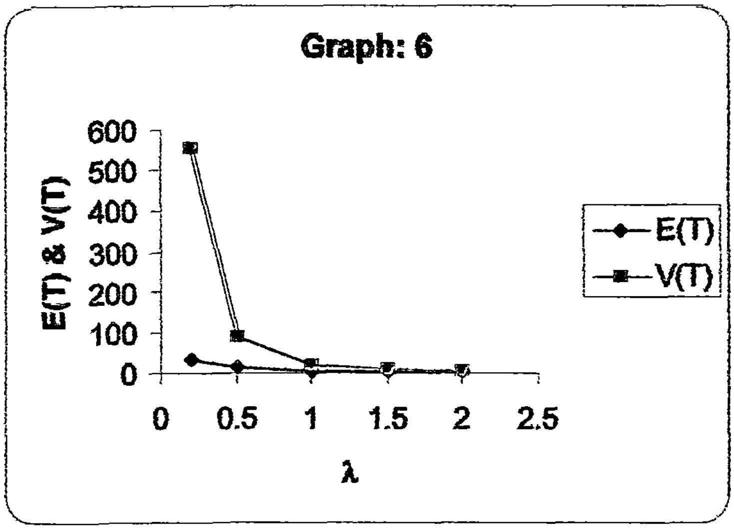
Case (vi)

Table: 6

k, θ, λ are also fixed. λ is allowed to vary.

K = 10, C = 1.5, θ = 1.0

λ	E(T)	V(T)
0.2	36.25	555.4
0.5	14.65	88.23
1	7.33	22.15
1.5	4.88	9.8
2	3.66	5.51



For the variance in λ, θ and C similar interpretations as in the case of first order statistic can be given.

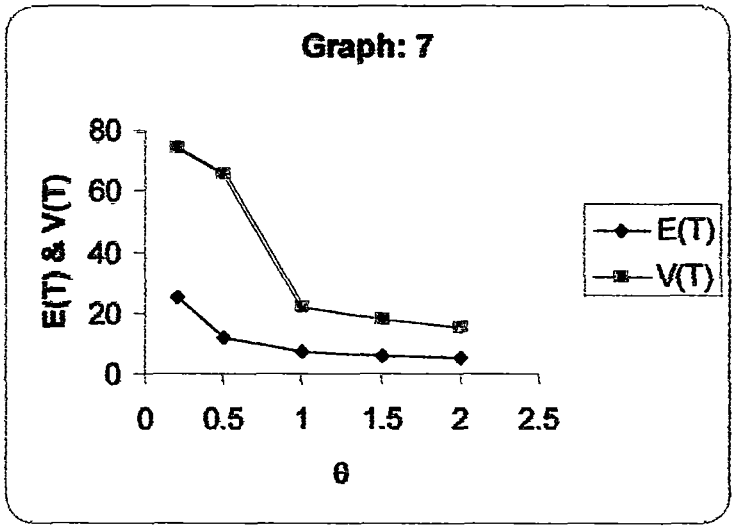
Case (vii)

Table: 7

k, θ, λ are also fixed. θ is allowed to vary.

K = 10 fixed c = 1.5 λ = 10

θ	E(T)	V(T)
0.2	24.90	73.82
0.5	11.72	65.49
1.0	7.33	22.06
1.5	5.86	17.53
2.0	5.12	15.13



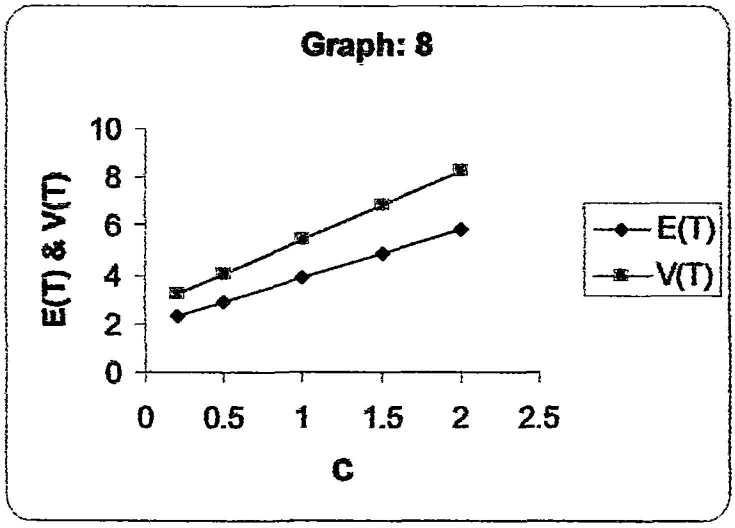
Case (viii)

Table: 8

k, θ, λ are also fixed. C is allowed to vary.

K = 10 θ = 1.0 λ = 1.5

C	E(T)	V(T)
0.2	2.34	3.21
0.5	2.93	4.03
1.0	3.91	5.43
1.5	4.88	6.85
2.0	5.86	8.29



NOTE:

1. It may be observed that in this model initially it is assumed that there are exactly 'K' occasions of policy decisions in (o,t). The interarrival times between the decisions epochs are arranged in increasing order of magnitude such that $U_{(1)} \leq U_{(2)} \ldots, U_{(k)}$ and hence $U_{(k)}$denotes the K^{th} order statistic. As K increases the length of the K^{th} interarrival time in order also increases. Hence the E(T) is bound to go up as K increases, and it depends upon the other parameters also.
2. It is a interesting note that in all this model as the value of θ, which is the parameter of the threshold distribution, which follows exponential has an increases. The values of E(T) and V(T) decreases. This is due to the fact that the random variable 'Y' denoting the threshold distribution follows exponential distribution with parameter θ, and so $E(Y)=\frac{1}{\theta}$. as 'θ' increases E(Y) decreases. Which means that the threshold level of wastage which is

in other words the permissible level of wastage becomes of smaller, and so expected time to reach the threshold which is denoted as E(T) decreases. In other words if permissible level of wastage is smaller, than the time to cross the threshold due to wastage on successive decision epochs become shorter.

Chapter – IV

A stochastic Model for Estimation of Expected Time to Recruitment under Correlated Wastage

CHAPTER – IV

A STOCHASTIC MODEL FOR ESTIMATION OF EXPECTED TIME TO RECRUITMENT UNDER CORRELATED WASTAGE

Introduction:

In any organization decisions regarding pay, perquisites and targets are revised periodically. Consequent to such decisions, there may be wastage of personnel, which leads to depletion of manpower. This kind of a phenomenon is very much pronounced in the case of marketing organisations. In the case of medical representatives in the pharmaceutical industry the exit of personnel is quite common. So also the case with software industry. The wastage may be due to

1. The unsatisfactory pay packages, but acceptable targets.

2. Satisfactory packages, unacceptable targets.
3. Both pay packages and targets are unsatisfactory.

The management is interested in reducing the level of wastage as much as possible.

It is of interest to study the wastage process and due to its impact what is the likely expected time to the breakdown or the likely time at which the fresh recruitments become necessary. In this context only the stochastic models are conceptualized and analyzed. In doing so the cumulative damage process and shock models are used. In the process of conceptualization of such models various assumptions are made, and suitable models are constructed.

One such assumption made is to treat the interarrival times of wastages at decision on epochs as identically and independently distributed random variables. S. Parthsarathy (2002) have taken a modification that the interarrival times between wastages are constantly correlated and exchangeable random variables. The results namely the estimation of E(T) and V(T) have been obtained.

In this chapter a different version of the model is discussed. In such a model it has been assumed that the interarrival times between wastage epochs are taken to be i.i.d random variables. But the assumption is that the amount of wastage at decision epochs are not i.i.d but constantly correlated. The following reasons can be seen as plausible.

1. If the wastage at a particular decision epoch is high then the organization takes suitable precautions so that the amount of wastage would be less at the ensuing epochs.

2. If the wastage at a particular epoch is high, it may have its own impact on the ensuing or succeeding epochs so that high rate of wastage may follow in the succeeding epochs.

Hence the model in which the amounts of wastages are correlated is conceptualized. In doing so the results by Gurland (1955) are used.

Assumptions:

1. There are 'k' epochs in (0, t) at which policy decisions are taken by the management.
2. At every decision epoch, there is a random amount of manpower wastage or depletion consequent to the decision taken.
3. The amounts of wastage at the k epochs are random variables. The amount of wastage is represented in terms of man hours.
4. The wastages are linear and cumulative.
5. If the total wastage crosses a level called the threshold, then the organization has to make recruitment of personnel since the cumulative manpower wastage is at a level which would make the organization to come to a breakdown point with regard to its activities.

Notations

X_i = a continuous a random variable representing the manpower wastage interms of man hours at each decision epoch, i = 1,2,...k, and p.d.f is g(.) and c.d.f G(.). X_i's are identically distributed constantly correlated, and exchangeable random variables.

Y = a continuous random variable denoting the threshold level of wastage and if the cumulative wastage crosses this level the recruitment is necessary. The p.d.f of Y is h(.) with c.d.f H(.).

U_i = a continuous a random variable denoting the interarrival times between decision epochs U_i, i = 1,2,...k and i.i.d with p.d.f f(.) and c.d.f F(.)

T = a continuous random variable denoting the time to breakdown, with p.d.f l(.) and c.d.f L(.)

$F_k(.)$ = k convolution of F(.)

$F_k^*()$ = Laplace transform of $F_k(.)$.

$F^*(.)$ = is the Laplace transform of $F(.)$

Results

It is a known result that

$$P\left[\sum_{i=1}^{k} Xi < Y\right] = \int_0^\infty g_k(x)\overline{H(x)}\,dx$$

It can also be proved that

$$P\left[\sum_{i=1}^{k} Xi < Y\right] = \int_0^\infty G_k(y)\; h(y) \quad dy$$

The survivor function is given as

$P[T > t] = S(t)$

$$= \sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right] \quad . \int_0^\infty G_k(y) h(y)\,dy \qquad ...(4.1)$$

It may noted that Y follows exponential with parameter θ. Hence the p.d.f of Y is $h(y) = \theta\, e^{-\theta y}$.

Since X_i's are assumed to be identical constantly correlated and exchangeable exponential random variables with parameter α, c.d.f of the partial sum S_k $= X_1 + X_2 + \dots X_k$ is given by Gurland (1955) as,

$$G_k(y) = (1-\rho)\sum_{i=0}^{\infty} \frac{(kp)^i}{(1-p+kp)^i} \frac{\varphi(k+1, Y/b)}{(k+i-1)!} \qquad \dots (4.2)$$

where

ρ = the constant correlation between any X_i and X_j, $i \neq j$.

$$\varphi(k+i, y/b) = \int_0^{y/b} e^{-z} \, z^{k+i-1} dz. \text{ and } b = \alpha\,(1-\rho)$$

Now substituting (4.2) and $h(y) = \theta\, e^{-\theta y}$ in (4.1) we get,

$$S(t) = \sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right] \quad \theta(1-\rho)$$

$$\int_0^{\infty} \sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^i} \quad \frac{\varphi(k+i, Y/b)}{(k+i-1)!} \; e^{-\theta y} \, dy \qquad \dots (4.3)$$

Now we consider

$$\int_0^{\infty} \varphi(k+i, y/b) e^{-\theta y} \, dy \qquad \dots (4.4)$$

$$\int_0^{\infty} \varphi(k+i, y/b) e^{-\theta y} \quad dy = \int_0^{\infty} \varphi(k+i, y/b) \, \frac{de^{-\theta y}}{-\theta}.$$

$$= \left[\varphi\left(k+i, \tfrac{y}{b}\right) \frac{e^{-\theta y}}{-\theta} \right]_0^{\infty} \quad \int_0^{\infty} \frac{e^{-\theta y} \varphi\left(k+i, \tfrac{y}{b}\right) d\left(\tfrac{y}{b}\right)}{-\theta} \left(k+i, \tfrac{y}{b}\right)$$

$$= \frac{1}{\theta} \int_0^{\infty} e^{-\theta y} \; e^{-y/b} \left(\tfrac{y}{b}\right)^{k+i-1} d\left(\tfrac{y}{b}\right)$$

$$= \frac{1}{\theta} \quad \frac{\Gamma_{k+i}}{(b\theta+1)^{k+i}}, \text{ on simplification} \qquad \dots (4.5)$$

since $\int_0^{\infty} e^{-x} \, x^{n-1} \, dx = \Gamma_n$.

and $\int_0^{\infty} e^{-ax} x^{n-1} dx = \frac{\Gamma_n}{a^n}$.

Substitution of (4.4) in (4.5) gives

$$\sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^i} \frac{1}{(k+i-1)!}, \quad \frac{1}{\theta} \frac{\Gamma_{k+i}}{(b\theta+1)^{k+i}}$$

$$= \frac{1}{\theta} \quad \sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^i (b\theta+1)^{k+i}}$$ since k + i = (k + i - 1)! ... (4.6)

Hence

$$S(t)=\sum_{k=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\theta(1-\rho)\quad\sum_{i=0}^{\infty}\frac{(k\rho)^i}{(1-\rho+k\rho)^i(b\theta+1)^{k+i}}$$

$$=\theta(1-\rho)\sum_{k=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\cdot$$

$$\frac{1}{(b\theta+1)^k}\quad\sum_{i=0}^{\infty}\left[\frac{(k\rho)}{(1-\rho+k\rho)(b\theta+1)}\right]^i$$

..... (4.7)

Now We have

$$\sum_{i=0}^{\infty}\left[\frac{(k\rho)}{(1-\rho+k\rho)(b\theta+1)}\right]^i=\sum_{i=0}^{\infty}x^i$$

$$= 1 + x + x^2 + \ldots\ldots$$

$$=(1-x)^{-1}=\left[\frac{(1-\rho+k\rho)(b\theta+1)}{(1-\rho+k\rho)(b\theta+1)-k\rho}\right]$$

on simplification

Hence

$$S(t)=\theta(1-\rho)\sum_{k=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]A_k$$

where

$$A_k=\frac{(1-\rho+k\rho)(b\theta+1)}{(b\theta+1)^k\left[(1-\rho+k\rho)(b\theta+1)-k\rho\right]}$$

Now

$$1 - S(t) = L(t) = P[T \leq t].$$

Hence

$$\mathcal{L}(t) = 1 - \theta(1-\rho)\left[\sum_{k=0}^{\infty} F_k(t)\, A_k - \sum_{k=0}^{\infty} F_{k+1}(t)\, A_k\right]$$

$$l(t) = \frac{d}{dt}\mathcal{L}(t)$$

$$= -\theta(1-\rho)\left[\sum_{k=0}^{\infty} f_k(t) A_k - \sum_{k=0}^{\infty} f_{k+1}(t) A_k\right]$$

... (4.8)

Taking Laplace transform of (4.8) we get

$$l^*(s) = -\theta(1-\rho) \quad \left[\sum_{k=0}^{\infty} f_k^*(s) \quad A_k - \sum_{k=0}^{\infty} f_{k+1}^*(s)\, A_k\right]$$

$$= -\theta(1-\rho)\left[\sum_{k=0}^{\infty}\left(f^*(s)\right)^k - \left(f^*(s)\right)^{k+1}\right]A_k \qquad \ldots(4.9)$$

Now

$$E(T) = \left.\frac{-d l^*(s)}{ds}\right|_{s=0}$$

Hence

$$E(T) = +\theta(1-\rho)\sum_{k=0}^{\infty}\left[k\left(f^*(s)\right)^{k-1}\frac{d}{ds}f^*(s) - (k+1)\left(f^*(s)\right)^{k}\frac{d}{ds}f^*(s)\right]A_k$$

...(4.10)

This is the most general form of E(T).

We consider the **Special case**

f(.) follows exponential distribution with parameter λ.

Hence

$$f^*(s) = \frac{\lambda}{\lambda + s}$$

$$\frac{d}{ds}f^*(s) = \frac{-\lambda}{(\lambda + s)^2}$$

$$(f^*(s))^{k-1} = \left(\frac{\lambda}{\lambda + s}\right)^{k-1}$$

Now in this case we have by the fact that

$$E(T) = \left.\frac{-d\,l^*(s)}{ds}\right|_{s=0}$$

E(T)

$$=\theta\,(1-\rho)\sum_{k=0}^{\infty}\left[-k\left(\frac{\lambda}{\lambda}\right)^{k-1}\frac{\lambda}{\lambda^2}+(k+1)\left(\frac{\lambda}{\lambda}\right)^{k}\left(\frac{\lambda}{\lambda^2}\right)\right]A_k$$

$$=\theta\,(1-\rho)\sum_{k=0}^{\infty}\left[\frac{-k}{\lambda}+\frac{(k+1)}{\lambda}\right]A_k$$

$$=\theta(1-\rho)\sum_{k=0}^{\infty}\left[\frac{1}{\lambda}\right]A_k$$

$$=\frac{\theta(1-\rho)}{\lambda}\sum_{k=0}^{\infty}A_k \qquad \ldots\ (4.11)$$

Substituting for A_k we have

$$E(T)=\frac{\theta(1-\rho)}{\lambda}\sum_{K=0}^{\infty}\frac{(1-\rho+k\rho)}{(b\theta+1)^{k-1}[(1-\rho+k\rho)\ \ (b\theta+1)-k\rho]}$$

$$\ldots(4.12)$$

Now

$$E(T^2)=\left.\frac{d^2 l^*(s)}{ds^2}\right|_{s=0}$$

$$\frac{d^2 l^*(s)}{ds^2}=-\theta(1-\rho)\sum_{k=0}^{\infty}\Big[\lambda(-1)(k+1)(\lambda+s)^{-(k+2)}+\lambda^k(-1)\ \ (k+1)(\lambda+s)^{-(k+2)}\,)$$

$$s\lambda^k(k+1)(k+2)(\lambda+s)^{-(k+3)}\Big]A_k$$

$$E\left(T^2\right)=\frac{d^2 l^*(s)}{ds^2}\bigg|_{s=0}=-\theta(1-\rho)\sum_{k=0}^{\infty}\left[(-1)\frac{\lambda^k}{\lambda^{k+2}}k+1+(-1)\frac{\lambda^k}{\lambda^{(k+2)}}(k+1)\right]A_k$$

$$=\theta\,(1-\rho)\sum_{k=0}^{\infty}\frac{2\lambda^k}{\lambda^{k+2}}\quad k+1\quad A_k$$

$$=2\theta\,(1-\rho)\sum_{k=0}^{\infty}\frac{k+1}{\lambda^2}A_k$$

$$E\left(T^2\right)=2\theta\,(1-\rho)\sum_{k=0}^{\infty}\frac{k+1}{\lambda^2}\quad\frac{(1-\rho+k\rho)}{(b\theta+1)^{k-1}(1-\rho+k\rho)\quad(b\theta+1)-k\rho}$$

$$\therefore\quad V(T)=E\left(T^2\right)-[E(T)]^2$$

$$V(T)=2\theta\,(1-\rho)\sum_{k=0}^{\infty}\frac{k+1}{\lambda^2}\quad\frac{(1-\rho+k\rho)}{(b\theta+1)^{k-1}\{(1-\rho+k\rho)\quad(b\theta+1)-k\rho\}}$$

$$-\left\{\frac{\theta(1-\rho)}{\lambda}\sum_{k=0}^{\infty}\quad\frac{(1-\rho+k\rho)}{(b\theta+1)^{k-1}\{(1-\rho+k\rho)(b\theta+1)-k\rho\}}\right\}^2$$

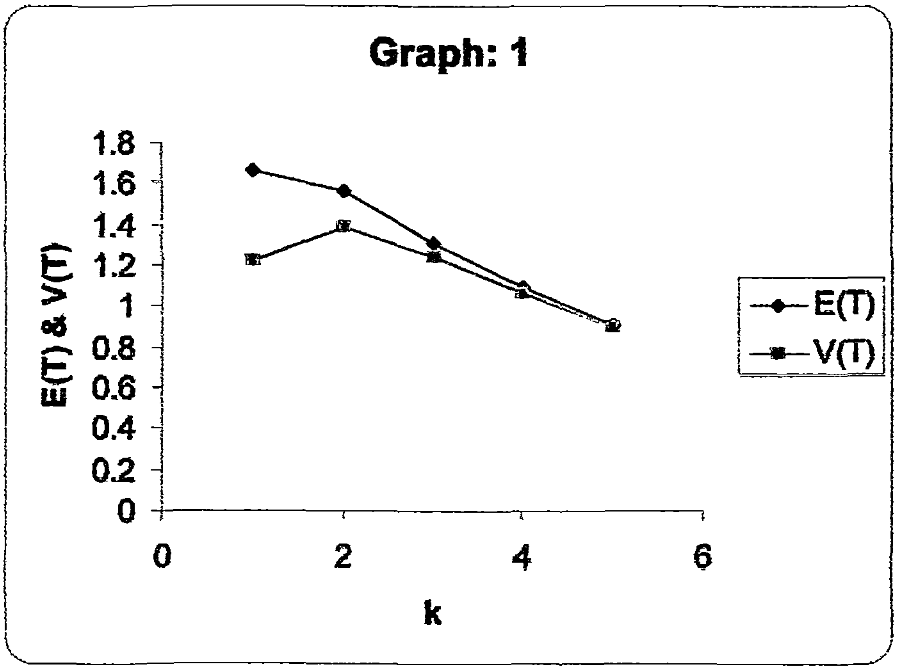
Here $b = \alpha(1 - \rho)$

Now the variations in E(T) and V(T) for the changes in the number of decision epochs is studied as under.

Table: 1

Let $\rho = 0.8$ $\alpha = 1.0$ $b = (0.2)$ $\theta = 1.5$ $\lambda = 1.2$

K	E(T)	V(T)
1	1.666666	1.222222
2	1.560333	1.387203
3	1.308294	1.239829
4	1.087946	1.057645
5	0.905617	0.891037



From the above table values of E(T) and V(T), it may be observed that as the number decisions 'k' in (o, t) increases, the expected time to breakdown of the system or the expected time to recruitment decreases gradually. It can also be inferred that after a certain value of 'k' the value of E(T) becomes zero there by implying that number of decisions should be kept limited.

The variance V(T) also becomes less pronounced there by indicating that as 'k' increases less of variations in E(T) is noted.

The tables are illustrated by graphs which explain the phenomenon clearly.

Chapter – V

A Stochastic Model for Study of Wastage when Preventive Strategies are used

CHAPTER – V

A STOCHASTIC MODEL FOR THE STUDY OF WASTAGE WHEN PREVENTIVE STRATEGIES ARE USED

Introduction:

Wastage in any organization is inevitable and cannot be altogether controlled and removed. But recruitment consequent to wastage would add to the expenses and would bring down the profits of the organization. One of the consequences of the wastage is that it leads to reduction of experienced and skilled personnel. This would in turn affect the productivity of the organization. Hence in many organizations the management aims at reducing wastage as much as possible. It is in this context some policy decisions are taken so as to offer some perquisites, incentives and promotion etc to motivate the personnel to continue in the organization itself. The propensity to leave the organization will be decreasing if suitable preventive strategies are adopted. This in other words it will mean that the rate of attrition will be brought down.

In many industrial and marketing organizations where the wastage is more pronounced the preventive strategies are adopted. In software industry now-a-days the switch over pattern of the personnel especially the highly skilled is very high. That is one of the reasons why the software personnel are paid a handsome pay and also very rich perquisites.

It may be noted that in some organizations (or) institutions while giving some promotions or other perquisites the organizations place some conditions such as execution of legal bonds not to leave the organization for a certain time period, or surrender of a few months of pay while leaving the organization. There are some kinds of preventive strategies to bring down the level of wastage..

It may be observed that immediately at each epoch of policy decision there may be wastage. At every such decision making epoch, the pros and cons of the policy decisions are well analysed and suitable preventive measures are also taken so as to reduce the amount of wastage. In this connection it is interesting to observe the following. The wastage process in any organization may be due to internal and external factors. The internal factors are purely from with in the organization such as lack of promotion opportunities, higher targets of achievement etc. They all come under the category of push factors. But there are some factors such as better opportunities in other organizations higher pay package etc, which come under the category of pull factors. Suitable strategies are used at decision making epochs to contain the amount of wastage, by containing the effect push and pull factors. But there may be some decision making epochs at which the preventive strategies may not be adopted or it may not be feasible to do so. On such occasions propensity to leave would be more and also the wastage. This would inturn affect the expected time to reach the threshold level at an earlier period (or) in a short duration.

In this chapter the impact of the preventive strategies on the value of E(T) and V(T) are analyzed. In doing so the wastage of manpower is taken to be a discrete random variable in the sense that the number of personnel leaving the organization at every decision epoch are considered.

Model - I

In this model we first derive the expression for E(T) under the assumption that at every occasion of announcement of policy decision, a certain number of persons leave the organization; which accounts for the wastage. The amount of wastage is represented as a discrete random variable and if the total amount of wastage (cumulative wastage) on 'k' occasions of policy decisions crosses the threshold level, then the recruitment becomes inevitable. The threshold itself is taken to be a discrete random variable. The model under question is derived as follows.

Assumptions of the model:

1. At every epoch of policy decision and its announcement a random number of persons leave the organization.
2. No recruitment is made and the wastage is allowed to cumulate till it reaches a threshold level which itself is a random variable.
3. The interarrival times between successive events of wastage are assumed to be identically and independently distributed.
4. The process of wastage is linear and additive

Notations:

X_i : a discrete random variable which represents the amount of wastage on the i^{th} occasion of policy announcement. X_i, $i = 1, 2, \ldots, k$ and i.i.d.

Y : a discrete random variable indicating threshold level of wastage beyond which the recruitment becomes necessary. Y is distributed as geometric with parameter θ.

U_i : a continuous random variable denoting the interarrival times between decisions. U_i's and i.i.d with p.d.f f(.) and d.f F(.)

p_n^i : $P[x_i = n]$ is the probability that 'n' persons leave the organization due to i^{th} decision.

h(s) : $\sum_{k=1}^{\infty} p_n^i s^n$, the p.g.f of X

$\phi(s)$: $\sum_{k=1}^{\infty} p_k s^k$, the p.g.f of Y

$\bar{\theta}$: Complement of θ (i.e) $\bar{\theta} = 1 - \theta$.

T : a continuous random variable denoting the time to reach the threshold. T has p.d.f $l(t)$ and the c.d.f is L(t).

Results:

If Y = j, then the threshold is j which means that the cumulative damage upto 'j' is permitted and no recruitment is necessary till then.

Now since Y follows geometric distribution

$$P_j = P[y = j] = (\bar{\theta})^{j-1}\theta$$

Hence

$$\varphi(s) = \sum_{k=1}^{\infty} p_k \; s^k$$

$$= \sum_{k=1}^{\infty} (\bar{\theta})^{k-1}\theta \; s^k$$

$$= s\,\theta \sum_{k=1}^{\infty} (s\,\bar{\theta})^{k-1}$$

$$= \frac{s\,\theta}{1 - s\,\bar{\theta}} \text{ on simplification} \qquad \ldots(5.1)$$

Now

$$P[X < Y] = \sum_{n=1}^{\infty} P[x_i = n] P[Y > n]$$

and

$$P[Y > n] = \sum_{i=1}^{\infty} P[Y = n + i]$$

Hence

$$P[X < Y] = \sum_{n=1}^{\infty} P[X = n] \sum_{i=1}^{\infty} P[Y = n + i]$$

Since Y follows geometric distribution

$$\sum_{i=1}^{\infty} P\,[y = n+i] = \sum_{i=1}^{\infty} (\bar{\theta})^{n+i-1}\,\theta \qquad \ldots(5.2)$$

$$= (\bar{\theta})^{n}\theta + (\bar{\theta})^{n+1}\theta + \cdots\cdots$$

$$= (\bar{\theta})^{n}\theta(1-\bar{\theta})^{-1}$$

$$= (\bar{\theta})^{n}$$

$$\therefore \quad P[X < Y] = \sum_{n=1}^{\infty} (\bar{\theta})^{n} P_n^1 = h(\bar{\theta}) \qquad \ldots(5.3)$$

This implies that the wastage will not cross the threshold level in any single decision epoch.

Now $P[X_1 + X_2 \cdots, X_k < Y] = P\left[\sum_{i=1}^{\infty} X_i < Y\right]$ gives the probability that the cumulative depletion or wastage of persons in 'k' decisions does not cross the threshold level.

Now $P\left[\sum_{i=1}^{k} X_i < Y\right] = [h(\bar{\theta})]^k$

since X_i are i.i.d. random variables.

Now we define S(t) as the survivor function which gives the probability that there are exactly 'k' decisions over the time interval or period (0, t) and that total wastage does not cross the threshold.

$S(t) = P[T > t]$

$= P$ [there are exactly k decisions in (0, t)]

$\times$ P[the cumulative wastage is less than Y]

Now by Renewal theory

P[there are exactly k decisions in (0, t)] $= [F_k(t) - F_{k+1}(t)]$

where $F_k(t)$ is the distribution function of $U_1+U_2+\ldots, U_k$ and is the k convolution of F(t) since U_i^1s and i.i.d.

Hence

$S(t) = P(T > t)$

$$= \sum_{k=0}^{\infty}\left[F_k(t) - F_{k+1}(t)\right] \left(h\left(\bar{\theta}\right)\right)^k$$

Now

$l(t) = 1 - S(t) = P(T < t)$

= Probability that the cumulative wastage crosses the threshold level before time 't'.

$$= \left[1 - h\left(\bar{\theta}\right)\right] \sum_{k=1}^{\infty}\left[h\left(\bar{\theta}\right)\right]^{k-1} F_k(t) \text{ on simplification} \quad \ldots(5.4)$$

We take the Laplace – Stieltje's transform on both sides and the L.H.S to denoted as L*(s)

$$L^*(s) = [1 - h(\bar{\theta})] \sum_{k=1}^{\infty} [h(\bar{\theta})]^{k-1} F_k^*(s)$$

By virtue of the fact that

$F_k^*(s) =$ Laplace transform of $F_k(t)$, can be written as

$\frac{1}{s} f_k^*(s)$,

and

$l^*(s)$ = L.S transform of $l(s)$ = $\frac{1}{s} L^*(s)$. gives

$$l^*(s) = [1 - h(\bar{\theta})] \sum_{k=1}^{\infty} [h(\bar{\theta})]^{k-1} f_k^*(s) \qquad \ldots(5.5)$$

and

$$f_k^*(s) = [f^*(s)]^k$$

since U_i's are i.i.d.

$$= [1 - h(\bar{\theta})] \sum_{k=1}^{\infty} [f_k^*(s)]^k [h(\bar{\theta})]^{k-1}$$

$$= [1 - h(\bar{\theta})] f^*(s) \sum_{k-1}^{\infty} [f^*(s)\, h(\bar{\theta})]^{k-1}$$

$$= \frac{[1 - h(\bar{\theta})] f^*(s)}{[1 - h(\bar{\theta})] f^*(s)} \text{ on simplification} \qquad \ldots(5.6)$$

Now

E(T) = expected time for the total wastage to cross the threshold

$$= \frac{-dl^*(s)}{ds}\bigg|_{s=0} = \mu_1^1$$

and

$$E(T^2) = = \frac{d^2 l^*(s)}{ds^2}\bigg|_{s=0} = \mu_2^1$$

$$V(T) = \mu_2^1 - \left(\mu_1^1\right)^2$$

which can be obtained from L*(s).

Special Case:

If it is assumed that the interarrival times U_i, follow exponential distribution with parameter c, we get

$$f^*(s) = \frac{c}{c+s}$$

$$\therefore l^*(s) = \frac{\left[1-h\left(\bar{\theta}\right)\right] \frac{c}{c+s}}{\left[1-h\left(\bar{\theta}\right) \frac{c}{c+s}\right]} \qquad \ldots(5.7)$$

Now

$$E(T)=\frac{1}{c\left[1-h\left(\bar{\theta}\right)\right]}$$

and

$$V(T)=\frac{1}{c^2\ \left[1-h\left(\bar{\theta}\right)\right]^2}$$ on simplification.

...(5.8)

A model of similar nature for a different problem of study and situation has been carried out by Sathiyamoorthi and Kannan (1998).

Model - II

In the above model we introduce the concept of preventive strategies and find its impact on E(T) and its variance, V(T).

It is assumed that some preventive strategy is adopted on each occasion of decision making and announcement. If the preventive measures are introduced then there will be no wastage or reduced level of wastage. If the preventive measures are not adopted the wastage will be more pronounced. Under these conditions the expressions for the E(T) and V(T) are obtained, by conceptualization of a suitable stochastic model.

Assumptions of the model:

1. Wastage of manpower occurs only at decision making epochs.
2. The threshold level is the upper limit of wastage. If the cumulative wastage crosses the threshold level then recruitment becomes necessary.
3. Wastage occurs or it is higher if only the preventive strategy is not used.
4. The probability of adopting the preventive strategy in a single decision making epoch is p and q = failure to use preventive strategy so that $p + q = 1$.
5. It is assumed that decision making is made in(0,t) at 'n' epochs, but there are only 'k' random epochs out of the 'n' decision epochs in which the preventive strategies are not adopted.

Notations:

In addition to the notations in model I, the following notations are also introduced.

W = A new continuous random variable representing the time interval between epochs, at which,the preventive strategies not followed, and has p.d.f g(.) and c.d.f G(.).

$F_n(.)$ = 'n' fold convolution of F(.), where F(.) is the c.d.f of the random variables U_i.

$G_k(.)$ = k fold convolution of G(.)

Results:

If may be observed that the wastage occurs (or) it is high only during such epochs at which the preventive measures are not used. So let us assume that there are k decision epochs at which the preventive strategies are not adopted.

In such a case, the survivor function is given by

S(t) = P (T > t)

$$= \sum_{k=0}^{\infty} V_k(t)\, P\left[\sum_{i=1}^{k} X_i < y\right]$$

Where $V_k(t) = G_k(t) - G_{k+1}(t)$ which gives, the probability that, there are exactly k decision epochs in (0,t) at which the preventive strategies are not used.

Hence

$$S(t) = \sum_{k=0}^{\infty} \left[G_k(t) - G_{k+1}(t)\right] \left[h\left(\bar{\theta}\right)\right]^k$$

Now,

$$L(t) = 1 - S(t) = P\,[T < t]$$

$$= 1 - \sum_{k=0}^{\infty} \left[G_k(t) - G_{k+1}(t)\right] \left[h\left(\bar{\theta}\right)\right]^k$$

$$= \left[1 - h\left(\bar{\theta}\right)\right] \sum_{k=0}^{\infty} \left[h\left(\bar{\theta}\right)\right]^{k-1} \left[G_k(t)\right]$$

on simplification.

Taking the Laplace Stieltjes transform of L (t) and on simplification

we get

$$L^*(s) = \frac{\left[1 - h\left(\bar{\theta}\right)\right] G^*(s)}{\left[1 - h\left(\bar{\theta}\right) G^*(s)\right]} \qquad \ldots(5.9)$$

Now, we have to obtain, the expression for G(w) in terms of F(w).

The expression for the distribution function of random variables 'W' which denotes, the interarrival times between the decision epochs at which the preventive strategies are not used, in terms of distribution function of U_i, the random variable which denotes the interarrival times between the decision epochs.

It may be noted that the probability of failure to use the preventive strategies q = 1- p.

$$\therefore G(w) = q \sum_{n=1}^{\infty} (\bar{q})^{n-1} F_n(w).$$

Where $\bar{q} = 1 - q = p$

The Laplace Stieltjes transform of G(w)

$$G^*(s) = \int_0^{\infty} e^{-st} dG(w)$$

$$= q \sum_{n=1}^{\infty} (\bar{q})^{n-1} \int_0^{\infty} e^{-st} dF_n(w)$$

$$= q \sum_{n=1}^{\infty} (\bar{q})^{n-1} F_n^*(s)$$

$$= q \sum_{n=1}^{\infty} (\bar{q})^{n-1} [F^*(s)]^n = qF^*(s) \sum_{n=1}^{\infty} (\bar{q}F^*(s))^{n-1}$$

since $U_i^{'}s$ and i.i.d.

Hence

$$G^*(s) = \frac{q\,F^*(s)}{1-\bar{q}\,F^*(s)} \text{ on simplification} \qquad \ldots(5.10)$$

Now substituting (5.10) in (5.9) we get

$$l^*(s) = \frac{\left[1-h\left(\bar{\theta}\right)\right] q\,F^*(s)}{1-\left[\bar{q}+h\left(\bar{\theta}\right)q\right] \; F^*(s)}$$

The mean time to cross threshold can be obtained by using fact that

$$\mu_1^1 = E(T) = \left.\frac{-dL^*(s)}{ds}\right|_{s=0}$$

$$\mu_2^1 = E\left(T^2\right) = \left.\frac{d^2L^*(s)}{ds^2}\right|_{s=0}$$

Using the above V(T) can be obtained as

$$V(T) = \mu_2^1 - \left(\mu_1^1\right)^2$$

Now

$$\left.\frac{dl^*(s)}{ds}\right|_{s=0} = \left[1-h\left(\bar{\theta}\right)\right]q \cdot \left(\frac{dF^*(s)}{ds}\right)_{s=0} \left[1-\left(\bar{q}+h\left(\bar{\theta}\right)q\right)F^*(s)\right]^{-1}$$

$$+\left[1-\left(\bar{q}+h\left(\bar{\theta}\right)q\right)F^*(s)\right]^{-2} \cdot \left[\bar{q}+h\left(\bar{\theta}\right)q\right] = \left(\frac{dF^*(s)}{ds}\right)_{s=0}$$

$$\left[1-h\left(\bar{\theta}\right)\right]q\,F^*(s)$$

Note:

Since the Laplace stieltjes transform of L(t) G(t) and F(t) and taken L*(s) and G(s) and F*(s) and similar to l*(s), g*(s) and f*(s) respectively.

Hence for e.g., $\mu_1^1 = E(T) = \left.\frac{-dL^*(s)}{ds}\right|_{s=0}$

$$= \left.\frac{-dl^*(s)}{ds}\right|_{s=0}$$

When even Laplace shieltjes transform is taken for L(s) have l*(s) = L*(s)

Now since

$$\left.\frac{-dF^*(s)}{ds}\right|_{s=0} = \mu_u, \text{ Where } \mu_u = E(U)$$

$$\frac{-dL^*(s)}{ds} = [1-h(\bar{\theta})]q \quad \mu_u \quad [1-(\bar{q}+h(\bar{\theta})q)]^{-1}$$

$$+[1-(\bar{q}+h(\bar{\theta})q)]^{-2} \quad [\bar{q}-h(\bar{\theta})q] \quad \mu_u \quad [1-h(\bar{\theta})]q$$

since $F^*(s)|_{s=0} = 1$

$$= \frac{[1-h(\bar{\theta})]q \quad \mu_u}{q[1-h(\bar{\theta})]} \left\{1-\frac{[\bar{q}+h(\bar{\theta})q]}{.q[1-h(\bar{\theta})]}\right\}$$

$$= \mu_u \left\{\frac{q = q\,h(\bar{\theta}) + \bar{q} + h(\bar{\theta})q}{q\ [1-h(\bar{\theta})]}\right\}$$

$$=\mu_u\left\{\frac{q=1-q}{q\left[1-h\left(\bar{\theta}\right)\right]}\right\}$$

$$=\mu_u\left\{\frac{1}{q\left[1-h\left(\bar{\theta}\right)\right]}\right\} \text{ on simplification}$$

$$E(T)=\mu_{ta} \quad =\frac{\mu_u}{q\left(1-h\left(\bar{\theta}\right)\right)}$$

$$\text{where } \mu_{ta}=\left.\frac{-dL^*(s)}{ds}\right|_{s=0}$$

...(5.11)

where μ_{ta} = expected time to breakdown under alter

ness (or) when preventive strategy is adopted.

Similarly it can be proved that

$$V(T)=\frac{\left\{\frac{\sigma_u^2+\mu_u^2(\bar{q}/q)}{q}+\frac{\mu_u^2 \quad h\left(\bar{\theta}\right)}{\left[1-h\left(\bar{\theta}\right)\right]}\right\}}{\left[1-h\left(\bar{\theta}\right)\right]} \qquad \text{...(5.12)}$$

Special Cases:

If we assume that the interarrival times between decision epochs U_i as following exponential distribution with parameter λ then $E(U_i)=\mu_u=\frac{1}{\lambda}$ for all $i = 1,2,\ldots, k$.

Hence substituting in (5.11)

we have

$$E(T) = \frac{\mu_u}{q\left[1-h\left(\overline{\theta}\right)\right]} = \frac{1}{\lambda q'\left[1-h\left(\overline{\theta}\right)\right]}$$

We have assumed that q = probability of not taking any preventive strategy to contain the wastage in a single decision epoch.

If q = 1, then it is a sure event and hence no preventive strategy is adopted.

In such a case

$$\mu_u = E(T) = \frac{\mu_u}{q\left[1-h\left(\overline{\theta}\right)\right]} = \frac{1}{\lambda\left[1-h\left(\overline{\theta}\right)\right]} \qquad \ldots(5.13)$$

If preventive strategy is used with probability 'q' then

$$E(T) = \mu_{ta} = \frac{1}{\lambda q\left[1-h\left(\overline{\theta}\right)\right]} \qquad \ldots(5.14)$$

Since q < 1 it is seen from (5.12) and (5.13) that

$$\mu_{ta} > \mu_t$$

Which implies that the expected time to breakdown (or) recruitment is larger when preventive strategy is adopted in relation to the expected time to recruitment when the same is not adopted. This proves the effectiveness of using the preventive strategies.

Chapter – VI

A Stochastic Model using Geometric Process for interarrival Times between Wastages

CHAPTER – VI

A STOCHASTIC MODEL USING GEOMETRIC PROCESS FOR INTERARRIVAL TIMES BETWEEN WASTAGES

6.1 Introduction

In this chapter a stochastic model in which the interarrival times between wastage epochs are according to a geometric process is considered.

In chapter III a stochastic model has been considered and in that model it has been assumed that the interarrival times U_i, i = 1,2,...k between successive epochs of wastage can be arranged according to increasing order of magnitude. Hence the order statistics $U_{(1)}$ and $U_{(k)}$ being the first order statistic and k^{th} order statistic. Using the distributions of $U_{(1)}$ and $U_{(k)}$ the expected time to cross the threshold and its variance have been obtained.

In deriving the E(T) and V(T), the sequence of random variables U_i = i = 1,2,...k which forms a geometric process is considered. The interarrival times form a sequence of random variables which are either stochastically increasing (or) stochastically decreasing. In such a case the expression of E(T) and V(T) are obtained.

The purpose of using a geometric process for the interarrival times between wastage epochs can be explained as follows.

In many organizations, especially in marketing and software organizations the management resorts to revisions of page scales, perquisites etc very often. In view of the competing conditions and pull factors that exist in these areas of employment, the managements are rather compelled to take decisions successively with small time gap in between. Due to the ever changing environment in employment conditions the interarrival times between decisions (or) epochs of wastage tend to form a decreasing sequence of random variables. In a similar way the interarrival times can also form an increasing sequence of random variables, in view of the following observations.

The organization may have the environment in which the propensity of the personnel may decrease as the completed length of service (CLS) increases. Hence wastage rate would also show a declining trend. The pull factors in the given sector comprising many such organizations and industries may become less pronounced. Hence the decision making epochs may be with longer time intervals.

6.2 Definition of a geometric process

Let $\{\xi(t),\ t \geq 0\}$ be a counting process, and let $\{U_n,\ n = 1,2,\ldots\}$ be a sequence of non negative independent random variables. The distribution function of U_n is denoted as $F(a^{n-1}t)$ $n = 1,2,\ldots k$ were 'a' is a positive constant. If $T_n = \sum_{i=1}^{n} U_i, n \geq 1$, and if $\{\ \xi(t),\ t \geq n\} = \{T_n \leq$

t} ($t_x \geq 0$, n = 1,2,...), then the counting process { $\xi(t)$, t ≥ 0} is called a geometric process. Here T_n is called the n^{th} change time. U_n is called the n^{th} change interarrival time.

Alternatively, given a sequence of non negative random variables X_i, i = 1,2,3,......, if for some a > 0, the sequence {$a^{n-1} X_n$, n = 1,2,...} forms a renewal process then { X_n, n = 1,2,...} is called a geometric process and 'a' is the parameter of the geometric process. If a = 1 than the geometric process is an ordinary renewal process.

If a > 1 then {U_n, n = 1,2,...} is stochastically decreasing (i.e) $U_n >_{st} U_{n+1}$, n = 1,2,... If 0 < a < 1, then {U_n, n = 1,2,...} forms a stochastically increasing sequence.

If a = 1, then {U_n, n = 1,2,...} is a sequence of independent random variables with a common distribution function and the geometric process { $\xi(t)$, t

$\geq 0\}$ is the ordinary renewal process. It may be noted that since $U_i^{'}s$ forms a geometric process, the c.d.f of U_n can be written as $W_n(u) = F(a^{n-1}u)$

Now

$$F_1(t) = P(U_1 + U_2 \leq t)$$

$$= \int_0^t f_1(u) w_2(t-u) du$$

$$= \int_0^t f_1(u) \ F(a[t-u]) du$$

and

$$f_2(t) \quad = \int_0^t f_{n-1}(u) w_2(t-u) du$$

Also

$$L^*[f_2(t)] \quad = \quad a \ f^*(s) \ L^* \ [f\ (at)]$$

$$= \quad a \ f^*(s) \tfrac{1}{a} f^*[s/a]$$

$$= \quad f^*(s) f^*[s/a]$$

In the same way it can be proved that

$$f_n^*(s) = \prod_{k=1}^{n} f^*\left(s/a^{k-1}\right)$$

The above results are useful in developing the model discussed in this chapter.

6.3 Assumptions of the model

1. Wastages occur at the decision making epochs.
2. The wastages on successive decision epochs are linear and cumulative.
3. The interarrival times between decision epochs are random variables which forms a geometric process.
4. As and when the total wastage on successive epochs crosses the threshold level the breakdown is imminent. In other words recruitment becomes necessary.

6.4 Notations

X_i = a continuous a random variable denoting the wastage on the ith decision epoch, i = 1,2,...k, the p.d.f of X_i is g(.) and c.d.f G(.).

U_i = a continuous a random variable denoting the interarrival times between decision (or) wastage epoch, $i = 1,2,\ldots k$. The p.d.f of U_i is $f(.)$ and c.d.f $F(.)$.

$W_n(.)$ The c.d.f of U_n the n^{th} change interarrival time (i.e) n^{th} term of geometric process and $w_n(.)$ the p.d.f.

Y = a continuous random variable denoting the threshold and Y is exponential with parameter θ.

$F_n(.)$ = A the p.d.f of $\sum_{i=1}^{n} X_i$

T = a continuous a random variable denoting the time to cross the threshold and the p.d.f of T is $l(.)$ and c.d.f $L(.)$.

6.5 Results

It is a known result that

$$P\left[\sum_{i=1}^{k} Xi < Y\right] = \int_0^{\infty} g_k xe^{-\theta x} dx$$

$$= g_k^* \quad (\theta)$$

$$= \left[g^*(\theta)\right]^k$$

Since X_1's and i.i.d

Now the survivor function is given by

$$S(t) = P(T > t) = \sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right] [g^*(\theta)]^k \qquad \ldots(6.1)$$

It can be shown that

$$S(t) = 1 - [1 - g^*(\theta)] \sum_{k=1}^{\infty} F_k(t) \; [g^*(\theta)]^{k-1} \qquad \ldots(6.2)$$

Now

$$L(t) = 1 - S(t)$$

$$= [1 - g^*(\theta)] \sum_{k=1}^{\infty} F_k(t) \; [g^*(\theta)]^{k-1}$$

Hence

$$l(t) = [1 - g^*(\theta)] \sum_{k=1}^{\infty} f_k(t) \; [g^*(\theta)]^{k-1}$$

and

$$l(t) = [1 - g^*(\theta)] \sum_{k=1}^{\infty} F_k(t) \, [g^*(\theta)]^{k-1},$$

Similarly

$$l^*(s) = [1 - g^*(\theta)] \sum_{n=1}^{\infty} [g^*(\theta)]^{n-1} f_n^*(s) \qquad \ldots(6.3)$$

Since $U_i^{'}$s form a geometric process,

$f_n^*(s)$ can be written as,

$$f_n^*(s) = \prod_{k=1}^{n} f^*\left(s/_{a^{k-1}} \right),$$

Hence

$$l^*(s) = [1 - g^*(\theta)] \sum_{n=1}^{\infty} [g^*(\theta)]^{n-1} \prod_{k=1}^{n} f^*\left(s/_{a^{k-1}} \right) \qquad \ldots(6.4)$$

Now

$$E(T) = \left. \frac{-dl^*(s)}{ds} \right|_{s=0}$$

$$= -[1 - g^*(\theta)] \sum_{n=1}^{\infty} [g^*(\theta)]^{n-1} \frac{d}{ds} \prod_{k=1}^{n} f^*\left(s/_{a^{k-1}} \right)_{s=0} \qquad \ldots(6.5)$$

We have

$$\prod_{k=1}^{n} f^*\left(s/_{a^{k-1}} \right)$$

$$f^*(s) f^*(s/_a) f^*(s/_{a^2}) \cdots f^*(s/_{a^{n-1}}) \qquad \ldots(6.6)$$

Hence

$$\frac{d}{ds}\prod_{k=1}^{n} f^*\left(s/a^{k-1}\right)\Big|_{s=0}$$

$$= f^{*1}(0) + \frac{1}{a} \quad f^{*1}(0) + \frac{1}{a^2} f^{*1}(0) + \frac{1}{a^{n-1}} f^{*1}(0)$$

where

$$f^{*1}(0) = \frac{d}{ds} \quad f^*\left(s/a^{k-1}\right)\Big|_{s=0} \qquad \text{for each n}$$

Since

$$f^{*1}(0) = 1$$

where

$$f^{*1}(0) = \frac{d}{ds} f^*(s)\Big|_{s=0}$$

$$= f^{*1}(0)\left[1 + \frac{1}{a} + \frac{1}{a^2} + \cdots\cdots + \frac{1}{a^{n-1}}\right]$$

$$= f^{*1}(0)\left[\frac{1 - \frac{1}{a^n}}{1 - \frac{1}{a}}\right],$$

$$= f^{*1}(0)\left[\frac{(a^{n-1})}{(a-1)(a^{n-1})}\right] \qquad \ldots(6.7)$$

Using (6.7) in (6.5) we get

$$E(T) = -[1-g^*(\theta)]\sum_{n=1}^{\infty}[g^*(\theta)]^{n-1} f^{*1}(0) \quad \frac{(a^n-1)}{(a-1)a^{n-1}}$$

$$= \frac{-[1-g^*(\theta)]}{(a-1)} f^{*1}(0)\sum_{n=1}^{\infty}\left(\frac{[g^*(\theta)]}{a}\right)^{n-1}(a^n-1)$$

$$= \frac{-[1-g^*(\theta)]}{(a-1)} f^{*1}(0)a\sum_{n=1}^{\infty}[g^*(\theta)]^{n-1} - \left(\frac{[g^*(\theta)]}{a}\right)^{n-1}$$

$$= \frac{f^{*1}(0)a}{[a-g^*(\theta)]} \qquad \ldots(6.8)$$

It can be proved that

$$E(U_1) = -f^{*1}(0)$$

Hence

$$E(T) = E(U_1)\frac{a}{[a-g^*(\theta)]} \qquad \ldots(6.9)$$

Similarly

$$E(T^2) = \left.\frac{d^2 l^*(s)}{ds^2}\right|_{s=0}$$

$$= [1-g^*(\theta)]\sum_{n=1}^{\infty}[g^*(\theta)]^{n-1}\frac{d^2}{ds^2}\prod_{k=1}^{n} f^*\left.\left(\frac{s}{a^{k-1}}\right)\right|_{s=0}$$

Now

$$\frac{d^2}{ds^2}\prod_{k=1}^{n} f^*\left.\left(\frac{s}{a^{k-1}}\right)\right|_{s=0}$$

$$\frac{d^2}{ds^2}\left(f^*(s)\ f^*(s/a)\ f^*\left(\frac{s}{a^{k-1}}\right)\right) \qquad \text{...(6.10)}$$

It can be shown that

$$\frac{d^2}{ds^2}\prod_{k=1}^{n} f^*\left(\frac{s}{a^{k-1}}\right)\Bigg|_{s=0},$$

$$= f^{11}(0)\left[1+\frac{1}{a^2}+\frac{1}{a^4}+\cdots+\frac{1}{a^{2(n-1)}}\right]$$

$$+\left[f^{*1}(0)\right]^2\left[\frac{1-1/a^n}{1-1/a}-1\right] \quad +\ldots\ldots\ldots$$

$$+\frac{1}{a^{n-1}}\left[f^{*1}(0)\right]^2\left[\frac{1-1/a^n}{1-1/a}-\frac{1}{a^{n-1}}\right] \qquad \text{..(6.11)}$$

$$=\left[f^{11}(0)-\left(f^{*1}(0)\right)^2\right]\frac{\left(a^{2n}-1\right)}{\left(a^2-1\right)a^{2(n-1)}}$$

$$=\left[\left(f^{*1}(0)\right)^2\right]\frac{\left(a^{2n}-2a^n+1\right)}{(a-1)^2a^{2(n-1)}} \qquad \text{...(6.12)}$$

on simplification and $f^{11}(0)$ denotes the second order derivative.

Hence

$$E\left(T^2\right)=\left[1-g^*(\theta)\right]\sum_{n=1}^{\infty}\left[g^*(\theta)\right]^{n-1},$$

$$\left[f^{*11}(0)-\left(f^{*1}(0)\right)^2\right]\frac{\left(a^{2n}-1\right)}{\left(a^2-1\right)a^{2(n-1)}} \qquad \text{...(6.13)}$$

It is seen that

$$\left[f^{*11}(0)-\left(f^{*1}(0)\right)^2\right]=\sigma^2_{U_1}$$

which is nothing but the variance of the random variable

U_1 and $\quad [E(U_1)]^2=[f^{*1}(0)]^2$

Hence

$$E\left(T^2\right)=[1-g^*(\theta)]\,\sigma^2_{U_1}\sum_{n=1}^{\infty}[g^*(\theta)]^{n-1}\frac{\left(a^{2n}-1\right)}{\left(a^2-1\right)a^{2(n-1)}}$$

$$+[1-g^*(\theta)]\,[E(U_1)]^2\sum_{n=1}^{\infty}[g^*(\theta)]^{n-1}\frac{\left(a^{2n}-2\,a^n+1\right)}{\left(a^2-1\right)a^{2(n-1)}}$$

$$\ldots (6.14)$$

Using the above results we get

$$V(T)=\frac{\left(a^2\sigma^2_{U_1}\right)}{\left(a^2-g^*(\theta)\right)}+[E(U_1)]^2\left[\frac{a^2}{(a-1)^2}-\frac{[1-g^*(\theta)]a^2}{(a-1)^2[a-g^*(\theta)]}\right]$$

$$+\frac{a^2[1-g^*(\theta)]}{(a-1)^2[a-g^*(\theta)]}-\frac{a^2}{(a-1)^2} \qquad \ldots(6.15)$$

$$=\frac{a^2\sigma^2_{U_1}}{\left(a^2-g^*(\theta)\right)} \quad \text{on simplification}$$

As a special case we consider g(.) is distributed as exponential with parameter λ.

Hence

$$g^*(\theta) = \frac{\lambda}{\lambda+\theta}$$

Hence

$$E(T)=E(U_1)\frac{a}{\left(a-\dfrac{\lambda}{\lambda+\theta}\right)}$$

$$=E(U_1)\frac{a(\lambda+\theta)}{(a(\lambda+\theta)-\lambda)}$$

Similarly

$$V(T)=\frac{a^2\sigma^2_{U_1}}{\left(a-\dfrac{\lambda}{\lambda+\theta}\right)} \quad V.(u_1)$$

i.i If we assume that Ui ~ exp (μ) then

$$E(U_1)=\frac{1}{\mu}$$

$$V(U_1)=\sigma^2_{U_1}\,\frac{1}{\mu^2}$$

Hence

$$E(T)=\frac{a}{\mu\left(a-\frac{\lambda}{\lambda+(\theta)}\right)}=$$

$$V(T)=\frac{a^{2}}{\mu^{2}\left(a-\frac{\lambda}{\lambda+(\theta)}\right)}$$

May be observed that, the mean and variance, namely the expected time to cross the threshold as well as its variances, namely E(T) and V(T) exit only that $[a=g*(\theta)]>0$. This is condition for feasible solution, it may be observed that the same condition is indicated by Jaisankar (2002).

Hence the Numerical Illustration is provided taking this constrained into consideration.

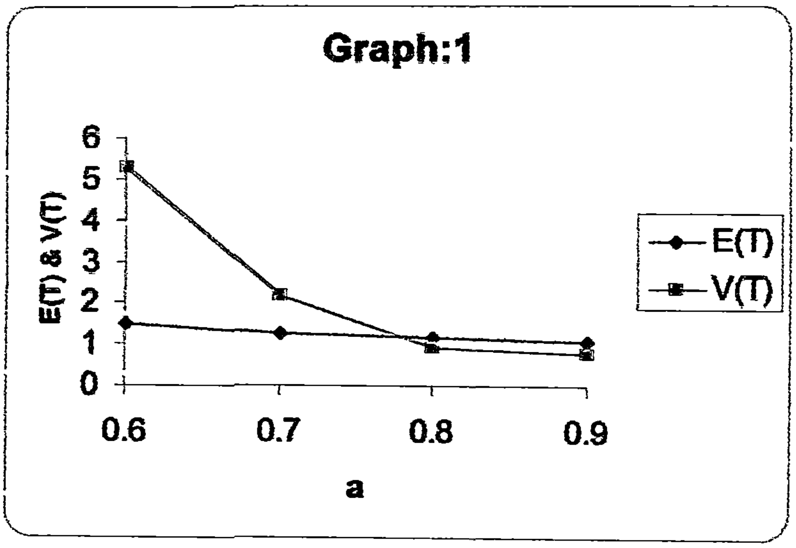
For a > 1 also the Numerical Illustration is provided given the values of μ, λ and θ.

Numerical Illustration

Table: 1

Let $\mu = 1.5$ $\lambda = 0.5$, $\theta = 1.0$ are all fixed

a	E(T)	V(T)
0.6	1.50	5.29
0.7	1.27	2.17
0.8	1.14	0.91
0.9	1.06	0.75

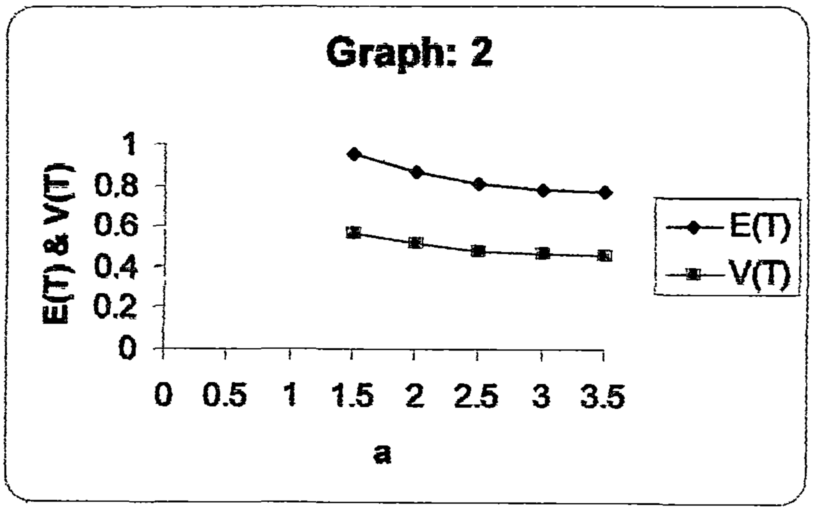


Case for the case when a > 1

Table: 2

Let $\mu = 1.5$ $\lambda = 1.0$, $\theta = 1.2$ fixed

A	E(T)	V(T)
1.5	0.95	0.56
2.0	0.86	0.51
2.5	0.81	0.48
3.0	0.78	0.47
3.5	0.77	0.46



It may be observed that if a > 1, then the U_i's, i = 1, 2,...n form a decreasing sequence. Hence the interarrival times between decision epochs will decrease. Hence the expected time to breakdown or recruitment will also decrease since the wastages would be more frequent.

Chapter – VII

Summary & Conclusions

CHAPTER - VII

SUMMARY AND CONCLUSIONS

On the basis of the various stochastic models discussed in the previous chapters the findings can be summarized as follows.

If the interarrival times between the successive decision making epochs are shorter in duration it shows that the expected time to recruitment also becomes shorter. If the length of the inter arrival times is elongated as in the case of K^{th} order statistics, the expected time to recruitment is slightly elongated. However the expected time to recruitment is dependent on the number of decision epochs over the period (0, t), the other parameters of the distribution, like the amount of wastage on every occasion, the threshold parameter and also the parameter of the distribution of the first order and for K^{th} order statistics.

If the interarrival times are i.i.d, but, if the amount of wastage on successive decision making epochs are correlated as discussed Chapter IV, it may be concluded that as, the number of decisions in (0, t) increases the expected time to breakdown (or) the expected time to recruitment decreases, which is quite reasonable since at every decision epoch there is loss of manpower and it get cumulated. The variance V(T) also decreases which implies less of oscillations or more of consistency with regard to the decline in the values of E(T).

It is interesting to note that the expected duration for recruitment on the average is longer if preventive strategies to avoid wastages are undertaken in comparison with the expected time to breakdown, if preventive strategies are not adopted. Hence it is interesting to observe the effectiveness of preventive measures. It is also a suggestion to the organizations to introduce some preventive strategies at decision making epochs, so that the need for frequent recruitments is avoided.

If the interarrival times are according to a geometric process, again, it is observed that the expected time to recruitment becomes a smaller, if a < 1. But if a > 1, the decrease in E(T) and V(T) becomes less pronounced. Again it implies that the interarrival times between decision making epochs are shorter then the expected time to recruitment becomes smaller. However all the parameters of the distributions involved influence the values of E(T) and V(T).

In order to make these models application oriented so that they will be of practical use in business and marketing organizations it becomes necessary to collect relevant statistical data regarding the different random variables that are involved in the model. The goodness of fit of the distribution should be investigated and then estimate of the parameters should be obtained using appropriate methods of estimation. In such a case effectiveness and practical utility of these models can be realized, and it would be of great use for the business and marketing organizations.

It is further suggested that these models can be used if the data set and observation available are truncated censored (or) both. In such cases the estimation of parameters of distributions involved will have to be carried out using the specific methods available for censored and truncated data. Also the models may have to be suitably described.

Appendix

APPENDIX

Answer for the questions by the external examiner:

The stochastic models developed in this thesis are primarily to measure the impact of wastage on the manpower availability in any organization. This kind of approach gives the likely time at which the organization would suffer from depletion of manpower availability and would make impossible to continue the operations. This would suggest the likely time at which the recruitment would be necessary.

In the present day context, the manpower models especially on the concept of wastage would help the organizations especially HRD division to plan the time points in future at which recruitment becomes imperative. In the software sector the depletion is more pronounced so that replacement and recruitment is more frequent. Hence the models serve as indicators and tools to decide the recruitment pattern. The preventive strategies to reduce the rate of attrition or wastage can also be adopted. Hence these models serve the purpose of determination of suitable strategies to make up loss of manpower. The planning to recruit people and to train then in advance help the organization to keep the manpower availability intact so that future work schedule is not affected. These models are especially relevant in the present day context in software sector. This kind of a phenomenon is slowly emerging in the structural construction section due to the increased demand for skilled workers like masons, carpenters, electricians etc.

It may be observed that the application of these models in practical situations is possible, provided the statistical methods are adopted suitably. The data collection is the most basic requirement. On the basis of the time series and cross sectional data it becomes imperative to find the estimates of the parameters of the probability distributions involved in these models. Then the test for goodness of fit of the assumed distribution must be carried out to find whether the distributions assumed coincide with the real life situations. Then the values of interest like E(T) can be worked out.

There are many other models which are used for the prediction of wastage in any manpower system. For example Cox's Regression Model is one which combines the Completed Length of Service (CLS) together with the covarites like spouse employment, pull factor from outside, etc to determine the level of individual's propensity to leave the job. It can be used to assess the level of attrition at different points of time in the future. Likewise the models which have been discussed in this thesis are also potential enough to have practical applications in real life situations. The practical utility of these models can also be verified using computer simulated data.

Questions:

Q1. How these models would obviously become relevant at the implementation stage of any emergent national policies for information manpower?

Answer:

In the field of information technology, the need for manpower in highly uncertain and it is essentially due to attrition or wastage. So the need for mathematical models is highly pronounced for the purpose of prediction and projections for the future. In doing so the national policy of any Government (or) any nation can contribute to the level of wastage (or) attrition.

For example:

If tax concessions are shown on the perquisites offered to the personnel in information sector, then it may attract more of persons and hence the shortage in manpower availability would be less. So it becomes necessary to use mathematical models of this type to estimate the likely time at which the manpower gap will be high, so that the normal functioning of the industries will not be possible. Therefore suitable preventive strategies and also precautionary measures can be implemented to come over the shortage of manpower.

Q2. How would you forecast the wastage?

Answer:

There are many methods which can be adopted for the forecast of wastage in any manpower system. It may be observed that there are many mathematical and statistical models which are used to predict the level of wastage and also the predict the amount of recruitments to be made. For this purpose a very simple model which is very much use is Cox's Regression Model in which both quantitative and qualitative factors which contribute to wastage are studied. The qualitative factors are like the employment of spouse, the availability of an alternative job, etc., which are called covariates of binary character, are used. This model can be used to predict the probability of an individual to leave the job at a given time point. Similarly the transition models based on Markov process also help in finding the size of attrition in the different grades in an organization. Using this model the level of wastage at future time point can be predicted.

Q3. In today's business world, capacities are predominantly determined by the available manpower resources. To meet the changing requirements and to keep a low operational cost, a timely adjustment of manpower is necessary. How will be models developed in the thesis help to improve the resource usage for government and industrial organizations?

Answer:

The models developed in this thesis are with the primary objective of finding the expected time at which the recruitment becomes necessary, because of wastage on successive occasions. It is also assumed that, in any organization (or) industry if the availability manpower is below, a particular level then the work schedule suffers. In other words if the amount of wastage crosses a particular leave called the threshold, and then the work schedule will suffer. Hence it becomes necessary to estimate the future time point at which recruitment will done. Also the preventive strategies can be implemented to reduce level of wastage. By such steps, the process of recruitment can be avoided (or) postponed. Since the recruitment is also costly it is better to avoid it as much as possible. These models help to find out, how the different measures taken up to avoid wastage help the organization to prolong the time for recruitment processes.

Q4. Manpower planning techniques have become an essential tool for human resource managers, especially in a climate of economic recession. Does the candidate have taken any effort towards tackling the problem through the models developed?

Answer:

It is quite true that the manpower planning models and techniques are very important tools for the human resources manager. In this climate of economic recession, it is very essential that the overall expenditure is any organization should be cut down. So the human research manager should try to find out the optimal level of manpower required in industries, and at the same time they should try to minimize the expenditure on recruitment and training of personnel. It is in this context, the mathematical models are used to predict, the manpower requirement, the level of wastage, and the time to recruitment etc. Such models help to carry out the usual activities. The models developed in this thesis can be definitely applied to tackle such problems. Even though the models developed in thesis are theoretical, they can be applied in practice and, the author of this thesis would take necessary steps for the implementation of these models in practical situation, in future studies.

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